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Undergraduate Instructional Laboratories

TO: Michael Baldwin June 13, 2017

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SUBJECT: Lab 4 – Blackbody Radiation

INTRODUCTION

The blackbody radiation experiment was performed at the Georgia Institute of Technology Mechanical Engineering Thermal Laboratory at the George W. Woodruff School of Mechanical Engineering on June 6, 2017. The objective of the lab was to verify the Stefan-Boltzmann Law, which is the governing equation for radiative heat transfer, by observing the change in net radiative heat transfer between two blackbodies and comparing calculated theoretical values to measured results. The blackbodies were subject to varying temperatures, diameters, and distance between them.

APPARATUS AND UNCERTAINTY

Apparatus. The data collected in this experiment required the use of two different systems mounted on a track ruler, which measured the distance between the systems. The system responsible for inputting heat into the experiment consisted of a blackbody cavity and a controller, specifically an IR 564/301 Blackbody System. The system responsible for measuring the net radiative heat transfer was the thermopile and the power meter, specifically a FieldMaxII-TO Laser Power Meter. Each of these systems consist of two separate instruments, but for this experiment, each system was considered to be one instrument, which is reflected in the uncertainty values. A caliper, specifically a Fowler ST133 0 to 20 cm with 0.02 mm marking, was used to measure the aperture dimeters in the blackbody cavity for data calculations and analysis. To establish the uncertainty of the caliper by comparison a gage block, specifically Weber Gage Div. – Starett Model RS45MA1, Grade 2, were utilized. To measure the ambient conditions a thermometer was used to measure the temperature, specifically a VWR General Purpose Glass Thermometer, Cat. #89095-598; - 20 °C to 110 °C with 1 °C markings, to measure the pressure a barometer was used, specifically O-N Ins. Aneroid Barometer; 500-780 mmHg with 5 mmHg markings. Table 1 lists the equipment used in this experiment and their associated uncertainties. The references for the uncertainty values are displayed below the table.

Table 1. Uncertainty of all utilized measurement devices.

Generic ID	Commercial ID	U_A	U_B	U_C
Thermopile and Power Meter	FieldMaxII-TO Laser Power Meter	0.03 mW[(4)]	0.19 mW[(4)]	0.19 mW
Blackbody Cavity and Controller	IR 564/301 Blackbody System	0.1°C[(3)]	0.2°C[(3)]	0.22°C
Track Ruler	-	0.05 cm[(1)]	0.018 cm[(6)]	0.053 cm
Caliper	Fowler ST133 0 to 20 cm w/ 0.02 mm marking	0.01 mm[(1)]	0.01 mm[(6)]	0.01 mm
Gage Blocks	Weber Gage Div. – Starett Model RS45MA1, Grade 2	N/A	0.2 µm[(5)]	0.2 µm
Thermometer	VWR General Purpose Glass Thermometer, Cat. #89095-598; -20 °C to 110 °C w/ 1 °C markings	0.5°C[(1)]	1°C[(7)]	1.1 °C
Barometer	O-N Ins. Aneroid Barometer; 500-780 mmHg with 5 mmHg markings	330 Pa[(1)]	N/A	330 Pa

[(1)] By inspection; [(2)] Zeroing; [(3)] Infrared Systems Development Corporation; [(4)] Coherent Laser Power Meter User's Manual; [(5)] Doirion T. and Beers (1995); [(6)] By comparison; [(7)] H-B Instrument Company (2009)

Uncertainty.

Three types of uncertainty are found for each apparatus. Type A uncertainty, U_A, is the uncertainty associated with error by the user. Type B uncertainty, U_B, is uncertainty associated with the device. Type C uncertainty, U_C, is found by relating Type A and Type B uncertainties, using Equation 1.

$$U_c = \sqrt{U_A^2 + U_B^2} \tag{1}$$

Type A Uncertainty.

The U_A of the track ruler, caliper, thermometer, and barometer is are 0.05 cm, 0.01 mm, 0.5°C, and 330 Pa respectively, determined by taking half of the smallest graduation. The U_A of the thermopile and power meter is 0.03 mW, determined by taking 0.1% of the full scale reading. The maximum full scale used in data collection was 30 mW. The U_A of the blackbody cavity and controller is 0.1°C, determined by the temperature resolution specifications found in the blackbody cavity specifications sheet published by Infrared Systems Development Corporation.

Type B Uncertainty.

The U_B of the blackbody cavity and controller, gage blocks, and thermometer is 0.2°C, 0.2 µm, and 1°C respectively, determined by the uncertainty data provided by the respective manufacturers: Infrared Systems Development Corporation, Doirion T. and Beers (1995), and H-B Instrument Company (2009). The U_B of the thermopile and power meter is 0.19 mW, determined by taking 1% of the max power meter reading which is specified by the device's manufacturer literature: Coherent Laser Power Meter User's Manual. The U_B of the track ruler and caliper is 0.018 cm and 0.01 mm respectively, calculated with Equation 2,

$$U_B = \sqrt{U_{B_{ref}}^2 + (measurement\ difference)^2} \tag{2}$$

where U_{Bref} is the type B uncertainty for the device being compared to the device in which the type B uncertainty is unknown and the measurement difference is the difference between the rated or listed measurement and the actual measurement confirmed by the secondary device. Please see the detailed caliper and ruler U_B calculations in Attachment 2 for calculation.

PROCEDURE

The following procedure was used to complete the experiment. Using these steps, several data points were recorded and calculations were made as discussed in the following section.

1. Record the ambient room temperature and pressure readings.

2. Measure and record the diameters of the four largest blackbody cavity aperture holes with a caliper. These aperture hole sizes will be referred to as 1-4 with 1 being the smallest hole and 4 being the largest hole.

3. Set the blackbody cavity controller to 300°C, move the thermopile 20 cm away, according to the track ruler, from blackbody cavity and set the blackbody cavity aperture hole to size 4. In all calculations from experimental data, add 4.4 cm to the distance between the thermopile and the blackbody cavity to account for the stand the thermocouple is mounted on.

4. Block the blackbody cavity aperture hole and zero the power meter.

5. Remove block and record the power meter Watt value and full-scale value.

6. Record the power meter Watt values and full-scale values for the aperture hole sizes 1-3, while keeping the thermopile at 20 cm.

7. Move the thermopile in 5 cm increments away from the blackbody cavity and record the power meter Watt and full-scale values of aperture sizes 1-4 at each increment until the thermopile reaches 50 cm.

8. Increase the temperature of the blackbody cavity controller to 400°C, reset the thermopile at 20 cm from the blackbody cavity, and wait 10 minutes for the blackbody cavity to warm up.

9. Block the blackbody cavity aperture hole and zero the power meter.

10. Repeat steps 5-7.

11. Repeat step 8 at 500°C and zero the power meter.

12. Repeat steps 5-7.

13. Repeat step 8 at 650°C to capture data points at an inconsistent temperature jump in hopes of more realistically verify the Stefan-Boltzmann Law and zero the power meter.

14. Repeat steps 5-7.

15. Repeat step 8 at 800°C and zero the power meter.

16. Repeat steps 5-7, but measure at 5 cm increments until 65 cm in order to establish whether or not the 50-65 cm range finds the Stefan-Boltzmann Law to be true.

DATA ANALYSIS AND FINDINGS

Part 1. Calculation of $\dot{Q}_{rad}$, the Radiative Heat Transfer

In order to calculate $\dot{Q}_{rad}$, the Stefan-Boltzmann law for radiative heat transfer is used. This is shown below in Equation 3,

$$\dot{Q}_{rad} = \sigma \varepsilon A_{s_1} F_{12} (T_1^4 - T_2^4) \tag{3}$$

where $\dot{Q}_{rad}$ is the rate of energy transferred through radiation in Watts, $\sigma = 5.67 \cdot 10^{-8}$ $Wm^{-2}K^{-4}$ is the Stefan-Boltzmann constant, $\varepsilon \approx 1$ is the emissivity, A_{s_1} is the area of the blackbody aperture in m^2, T_1 is the temperature of the blackbody in Kelvins, $T_2 = 294.85$ K is the ambient temperature of the surroundings, and F_{12} is the view factor between the thermopile and blackbody cavity aperture shown below in Equation 4.

$$F_{12} = \frac{1}{2}\left[S - \sqrt{S^2 - 4\left(\frac{D_2}{D_1}\right)^2}\right] \tag{4}$$

where D_1 is the diameter of the blackbody aperture in meters, $D_2 = 0.01100$ m is the diameter of Thermopile Radiometer, and S is calculated below in Equation 5,

$$S = 1 + \frac{4L^2 + D_2^2}{D_1^2} \tag{5}$$

where S is the variable in Equation 2, D_1 is the diameter of the blackbody aperture in m, $D_2 = 0.01100$ m is the diameter of Thermopile Radiometer, and L is the distance between the blackbody aperture and the Thermopile Radiometer.

Below is the equation used to solve for A_{s_1} in Equation 3,

$$A_{s_1} = \frac{\pi D_1^2}{4} \tag{6}$$

where A_{s_1} is the area of the blackbody aperture in m^2 and D_1 is the diameter of the blackbody aperture in m.

By using the above equations, the theoretical net radiative heat transfer is calculated by computing S, F_{12}, and $\dot{Q}_{rad}$ as seen in Attachment 1. Through the variation of D_1, L, and T_1, the experimental $\dot{Q}_{rad}$ is compared against the theoretical $\dot{Q}_{rad}$ to verify the Stefan-Boltzmann law in Equation 1.

Part 2. Error Analysis of $\dot{Q}_{rad}$, the Radiative Heat Transfer

Q_{rad} can be expressed as $Q_{rad} = f(T_1, T_2, D_1, D_2, L)$ so the error propagation analysis is carried out as Equation 7,

$$U_q = \sqrt{\left(\frac{\partial q}{\partial T_1} U_{T_1}\right)^2 + \left(\frac{\partial q}{\partial T_2} U_{T_2}\right)^2 + \left(\frac{\partial q}{\partial D_1} U_{D_1}\right)^2 + \left(\frac{\partial q}{\partial D_2} U_{D_2}\right)^2 + \left(\frac{\partial q}{\partial \mathrm{L}} U_{\mathrm{L}}\right)^2} \tag{7}$$

where U_q is the total combined uncertainty for the net radiation heat transfer, U_{T_1} is the uncertainty for the blackbody cavity temperature, U_{T_2} is the uncertainty for the laboratory ambient temperature, U_{D_1} is the uncertainty for the diameter of the blackbody cavity aperture, U_{D_2} is the uncertainty for the thermopile sensor diameter, and U_{L} is the uncertainty for the distance between the thermopile and the blackbody cavity. The value for U_{T_1} is the combined uncertainty for the blackbody cavity and controller, the value for U_{T_2} is the combined uncertainty for the thermometer, the U_{D_1} and U_{D_2} values are the combined uncertainty for the caliper, and the value for U_{L} is the combined uncertainty for the track ruler. These combined uncertainty values are computed in Attachment 3 and parameter uncertainty values can be found in Table 1.

Since the equation is complex, the partial derivatives with respect to each parameter is difficult to compute. Hence, the partial derivatives were found by other numerical methods. The partial derivative is estimated to equal a small change in the heat transfer value, with respect to that parameter. The small change used in the calculations, is the inherent uncertainty in that parameter value as detailed in Table 1.

The influence coefficients are found by applying Equation 8 to each parameter.

$$\frac{\partial Q}{\partial P} \approx \frac{\Delta q}{\Delta P} \tag{8}$$

where P is any of the parameters that Q depends on.

Attachment 4 shows a sample calculation for one parameter T_1. This calculation as repeated for each parameter to calculate the total uncertainty in the net heat transfer by radiation. The calculated values for the S, F_{12}, influence coefficients, relative uncertainties (experimental to theoretical), and total q uncertainty is calculated for the data point set of $T_1 = 21.7°C$, $T_2 = 800°C$, $D_1 = 0.490$ cm, $D_2 = 1.1$ cm, and $L = 20.0$ cm. These values were chosen to maximize the uncertainty to compute an upper bound for the random error.

The calculations were performed as presented in Attachment 3, and the results are summarized in the table below.

Table 2. Summary of final calculated uncertainty values from Attachment 3.

	Q_{model} (mW)	**EPA Uncertainty (mW)**	**Percentage of Uncertainty (%)**
T_1	2.7	0.0040	0.15
T_2	2.7	0.000085	0.003
D_1	2.7	0.11	4.09
D_2	2.7	0.049	1.8
L	2.7	0.13	5.0
Q	2.7	0.18	6.7

It can be concluded that the total combined uncertainty for the net radiation heat transfer is 0.18 mW. The largest uncertainty comes from the length measurements, while the uncertainty is relatively unaffected by the ambient laboratory temperature reading and the blackbody cavity aperture diameter measurements.

Part 3. Effect of varying temperature on heat loss by radiation

According to the Stefan-Boltzmann Law in Equation 3, the temperature and the heat transfer due to radiation are related by a 4th order polynomial. Hence, if everything else is kept constant, the data should fit the following curve in Equation 9,

$$\dot{Q}_{rad} = C_1 \cdot T_1^4 + C_2 \tag{9}$$

where $\dot{Q}_{rad}$ is the rate of energy transferred through radiation in Watts and T_1 is the temperature of the blackbody in Kelvins.

The data was filtered and plotted according to Attachment 5.

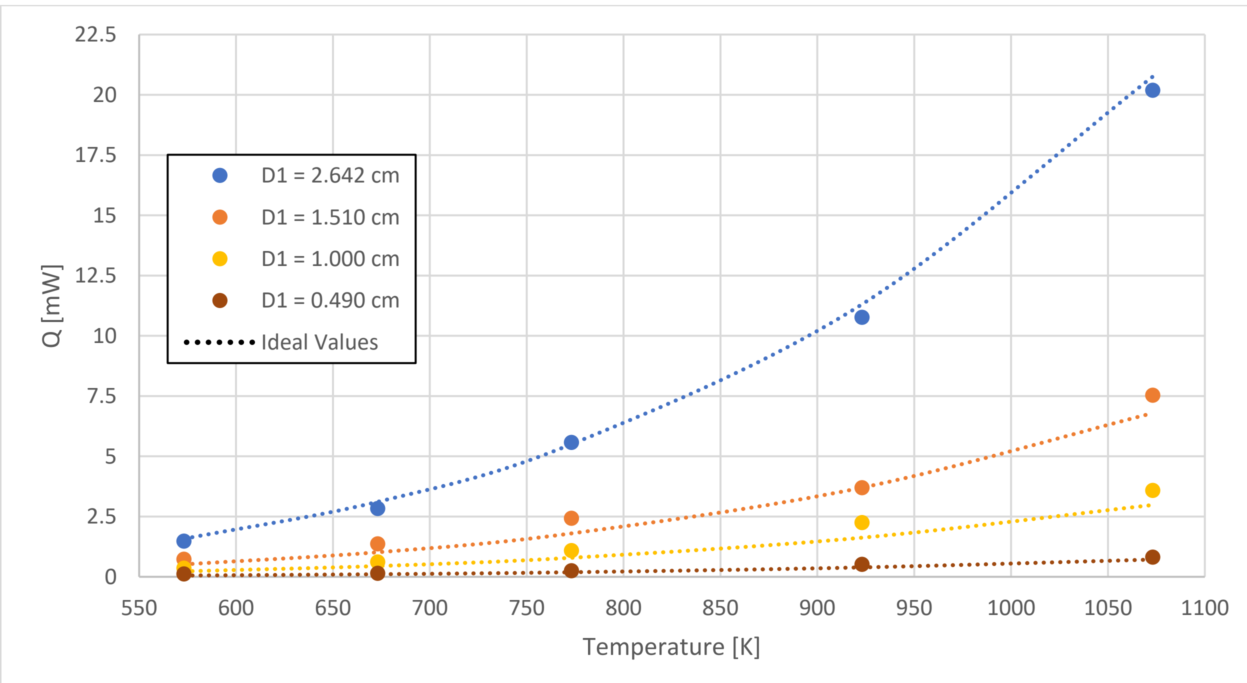

Figure 1. Plot of ideal and actual radiative power against temperature for four different lengths

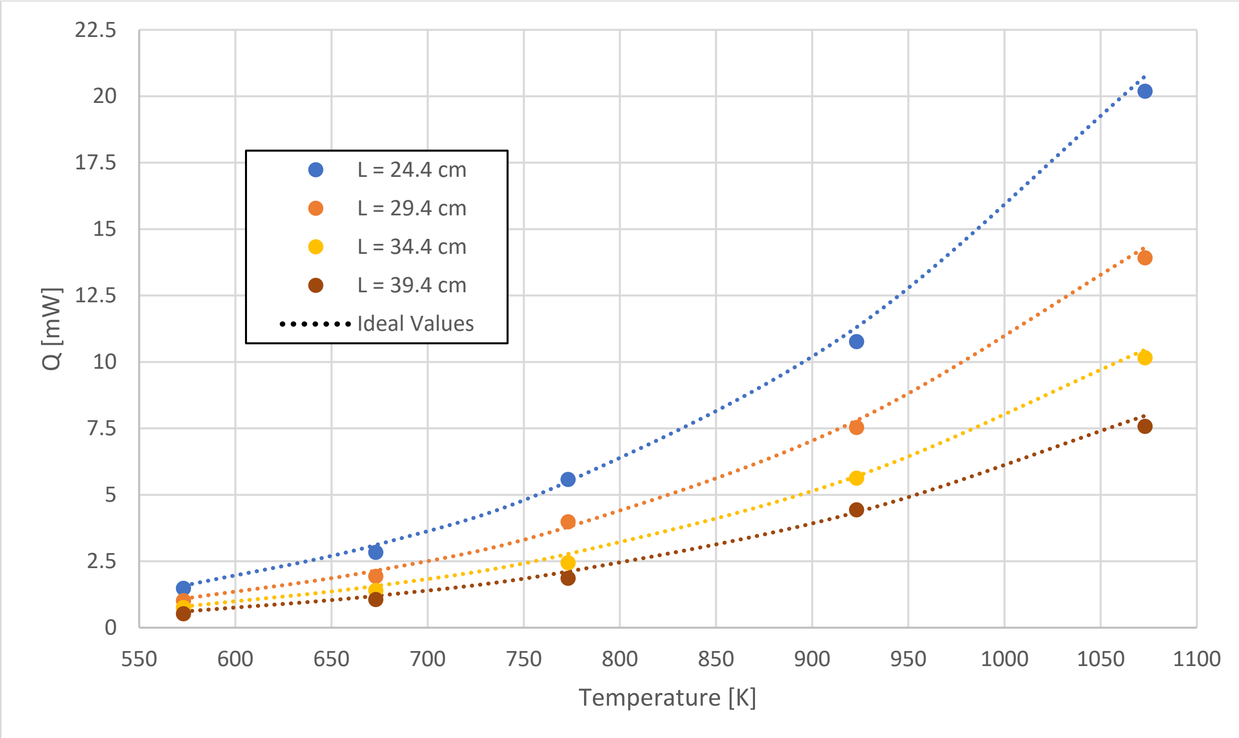

Figure 2. Plot of ideal and actual radiative power against temperature for different diameters

The figures above show the relationship that the temperature of the black body has with the radiative power. Based on Equation 3, the experimental data should fit a 4th order polynomial; however, since only 5 temperature values were taken, there are zero degrees of freedom, so an alpha risk cannot be calculated and any 4th order polynomial fit through these points will have an R^2 value of 1. Although there is not enough data to show a 4th order fit, it can be noted that the experimental data that was taken is very close to the ideal values calculated using the Stefan-Boltzmann equation.

From the data, the largest difference between experimental and ideal values for Q occurred at the largest temperature values. This is because when these data points were taken, there was a rush for time and the blackbody cavity's temperature was still fluctuating slightly. The power meter also gives a larger uncertainty at higher temperatures, which could also lead to greater error. For the remainder of the data, the error lies within the uncertainty of 0.18 mW. Attachment 5 shows the calculated RMS error for the figures above and they are 0.28 and 0.37 mW respectively. In addition to the fluctuations in temperature at higher readings, there were also great fluctuations in the power meter at low readings due to background radiation. This further contributed to the errors in the data. Hence, the few points that are outside the uncertainty are accounted for by systematic error, and the temperature therefore fits the 4th order polynomial as shown in Figures 1 and 2.

Part 4. Effect of varying blackbody cavity diameter on heat loss by radiation

The length and temperature were kept constant while varying diameter to produce the following graphs. (Raw data in Attachment 6).

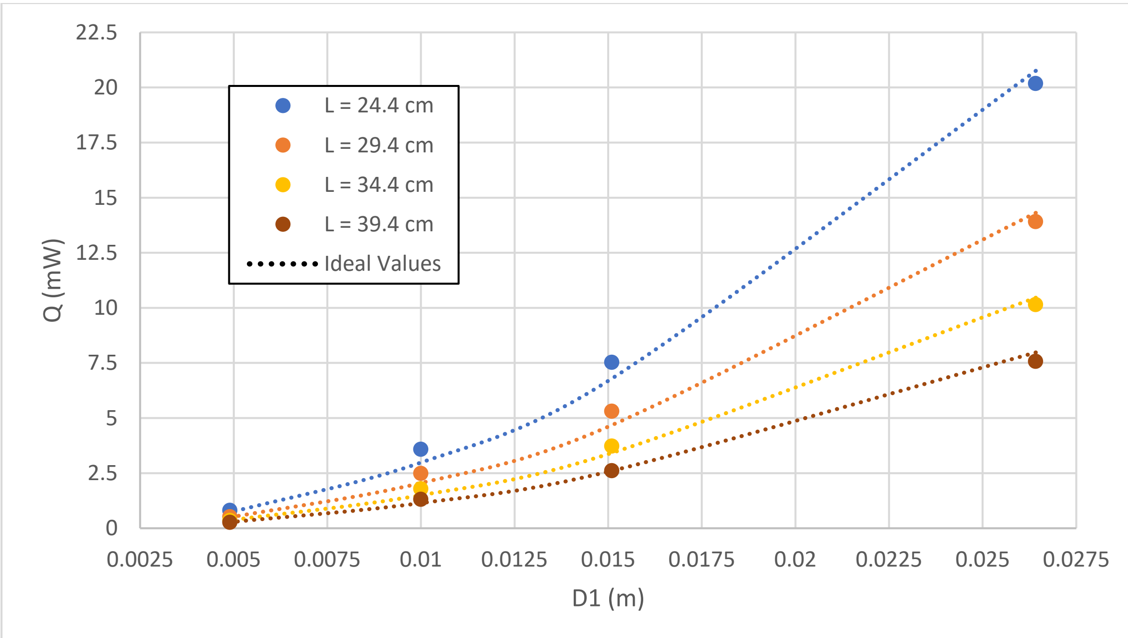

Figure 3. Plot of ideal and actual radiative power against diameter for four different lengths

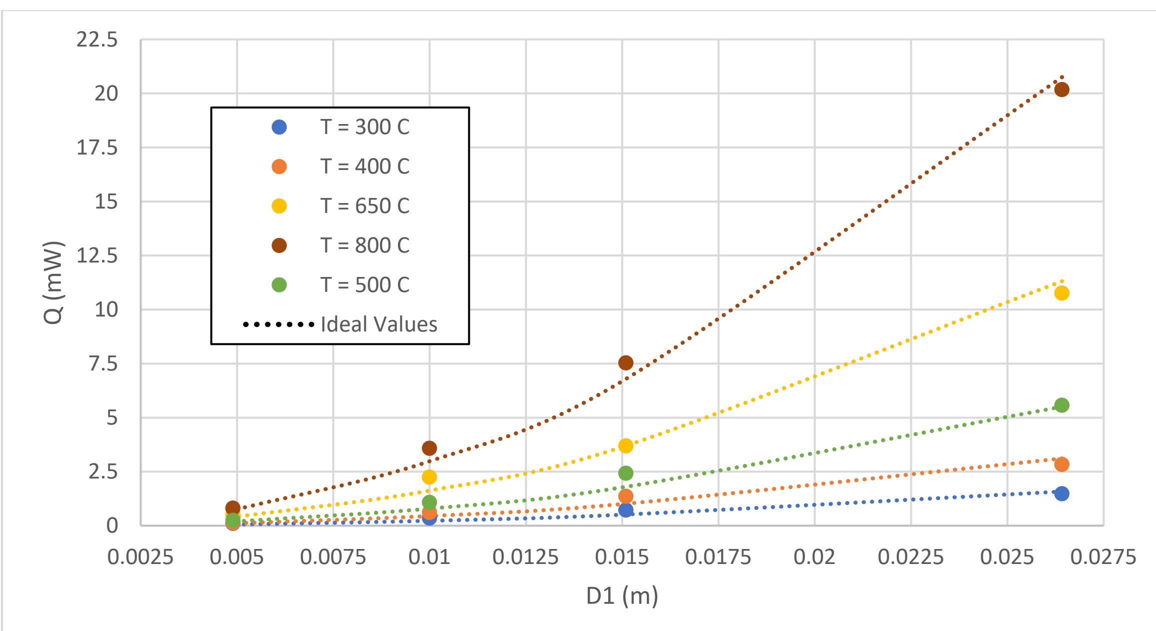

Figure 4. Plot of ideal and actual radiative power against diameter for different temperatures

By simplifying the Stefan-Boltzmann equation shown in Equation 3, it was determined that the radiative heat transfer should theoretically fit a second order polynomial in relation to diameter. Furthermore, attachment 6 shows that at a constant length, the view factor remains relatively constant; therefore, only the area should affect the radiative power at constant length and temperature, this explains the 2^{nd} order polynomial fit in Figures 3 and 4 for radiative power versus diameter.

Figures 3 and 4 show this relation of radiative power to diameter. Attachment 7 shows the regression of a 2^{nd} order polynomial fit through this data. Since the greatest P-value for the coefficients was 0.024 (which is less than 0.05), it passes the regression analysis test, and a 2^{nd} order polynomial is appropriate to fit the data. The R^2 value of nearly 1 also proves that this is an almost perfect fit. From this we can conclude that the correlation presented in the Stefan-Boltzmann law was verified experimentally.

The accuracy of the fit was determined by calculating the error between experimentally measured radiative heat transfer value and theoretical values. The RMS values were calculated as 0.39 and 0.41, as shown in Attachment 6. The high RMS value is due to some data points varying significantly from expected values. The greatest deviation was 0.73 mW which is much higher than the 0.18 mW expected uncertainty. This points to the existence of systematic errors.

The reason for this high error is once again from the outliers of some data points that have exceptionally large errors. These errors always occur at values with high radiative power. This can be attributed to the low resolution of the power meter at a higher full scale value. The full-scale value changes at 3 mW. This explains why all the values below 3 mW were within the 0.18 mW uncertainty. Above the 3 mW range, the full-scale is realized and the sensitivity is reduced by an order of 10. Hence, radiative heat transfer values above 3 mW are much less accurate.

Part 5. Effect of varying length between thermopile and blackbody on heat loss by radiation

The diameter and temperature were kept constant while varying length to produce the following graphs. (Raw data in Attachment 8).

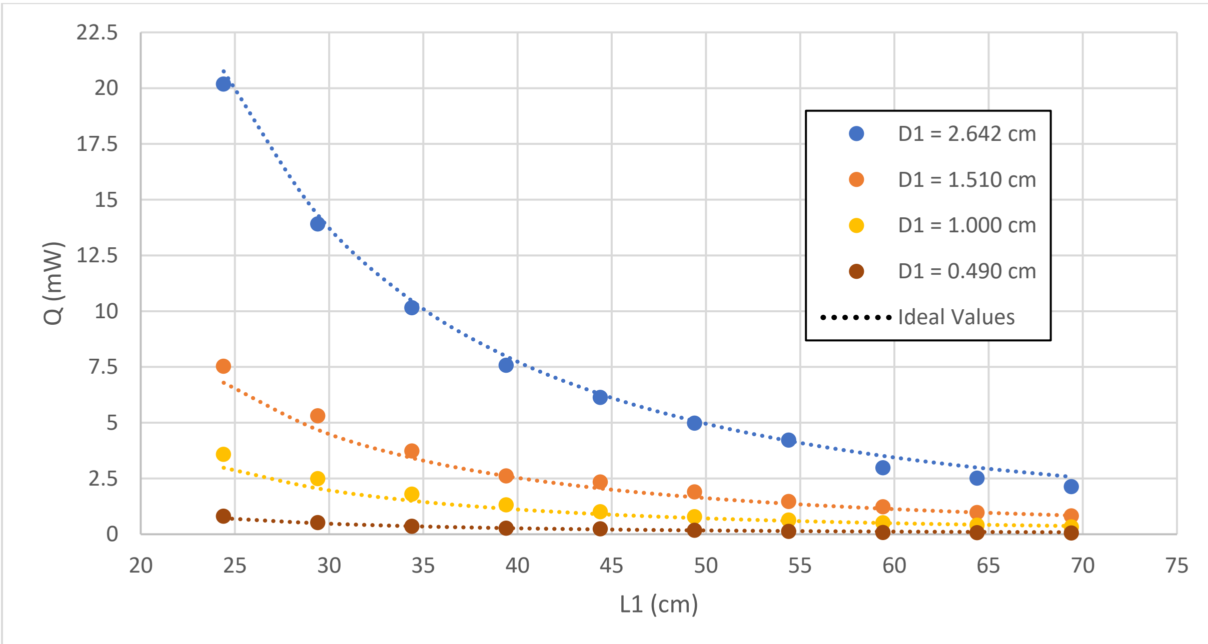

Figure 5. Plot of ideal and actual radiative power against length for four different diameters

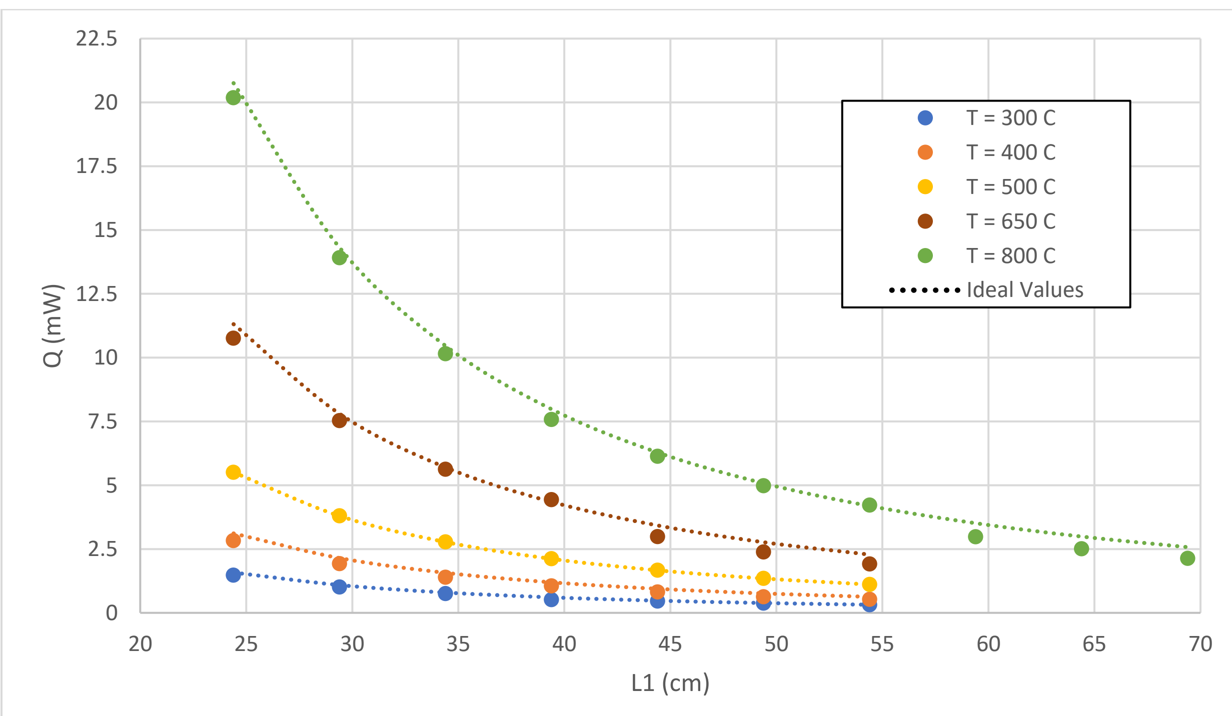

Figure 6. Plot of ideal and actual radiative power against length for four different temperatures

By simplifying the Stefan-Boltzmann equation shown in Equation 3, it was determined that the radiative heat transfer is inversely proportional to the square of the length, so it should theoretically fit an L_1^{-2} curve. Figures 5 and 6 show this relation of radiative power to length. Attachment 9 shows the regression of a $\frac{1}{L_1^2}$ fit through this data.

Since the greatest P-value for this linear fit was 3.86E-13 (which is far less than 0.05), this data can be classified as inversely proportional to the square of the length. The R^2 value of 0.998910023 also proves that this is an almost perfect fit. When calculating the RMS error of the figures shown above, the values calculated were 0.29 and 0.28, as shown in Attachment 8. The reason for this high error is once again from the outliers of some data points that have exceptionally large errors.

These errors always occur at values with high radiative power. This can be attributed to the low resolution at a higher full scale value. The full-scale value changes at 3 mW. This explains why all the values below 3 mW were within the 0.18 mW uncertainty. Since the P-values were so low and the R^2 values were large for the figures shown above, it can be concluded that the radiative heat transfer is inversely proportional to the square of the length with high confidence.

Part 6: Validation of the Stefan-Boltzmann Law

Given the Boltzmann relation in Equation 3, all the parameters that are being varied (L_1, D_1, T_1) can be grouped under one variable. This is shown in Equation 10,

$$\dot{Q}_{rad} = \sigma \varepsilon A_{s_1} F_{12}(T_1^4 - T_2^4) = \sigma C \tag{10}$$

where $C = \varepsilon A_{s_1} F_{12}(T_1^4 - T_2^4)$, $\sigma = 5.67 \cdot 10^{-8}$ is Stefan-Boltzmann constant and $\dot{Q}_{rad}$ is the radiative heat transfer in W.

For all 152 collected data points, the value of C was calculated and plotted on a single graph. According to the Stefan-Boltzmann law, the resulting graph should be linear and should have a slope equal to that of the Boltzmann constant.

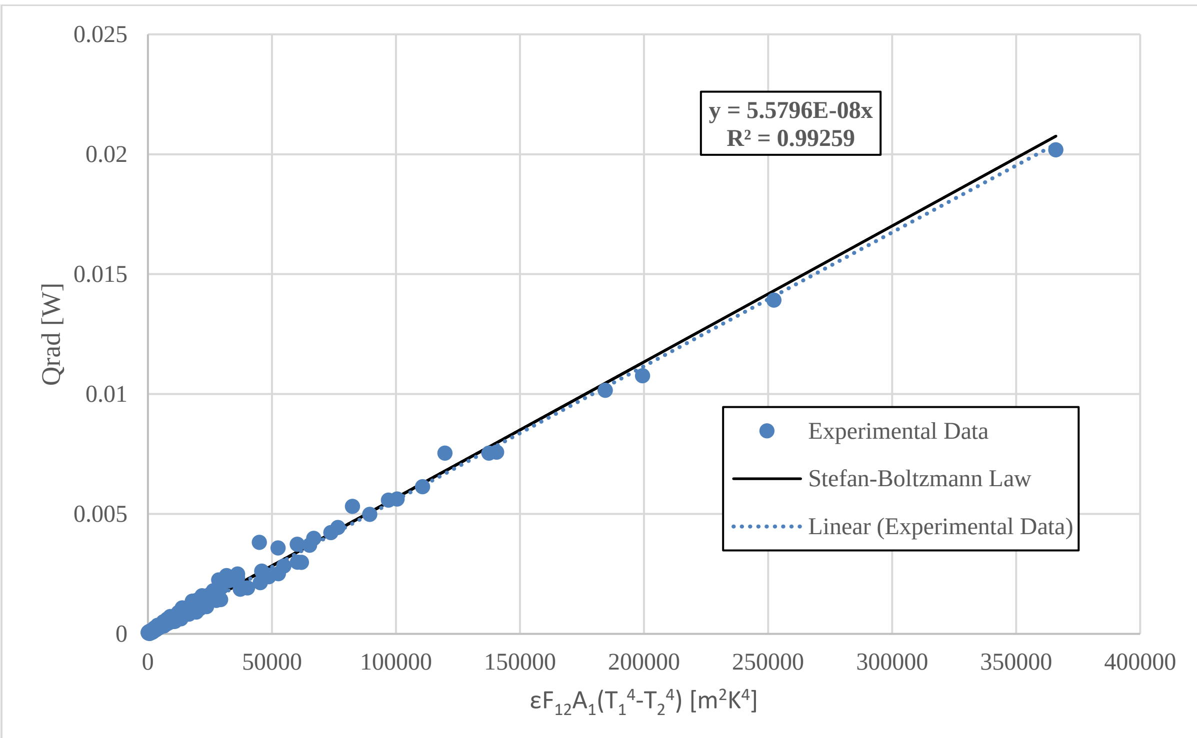

Figure 7. Radiative heat transfer plotted against all varying parameters

From the figure, above, the slope of a fitted trendline through the data points is $5.5796 \cdot 10^{-8}$. The linear trendline fits the data well. It has an alpha risk of $4.9 \cdot 10^{-172}$ and an R^2 value of 0.9942 (Attachment 10). This passes the regression analysis and concludes that a linear fit is appropriate for the data.

The slope of the data itself is very close to the Stefan-Boltzmann constant as can be seen in the figure. The difference is $9.04 \cdot 10^{-10}$. This slope is calculated by Equation 11,

$$m = \frac{\dot{Q}_{rad}}{C} \tag{11}$$

where $C = \varepsilon A_{s_1} F_{12}(T_1^4 - T_2^4)$ and $\dot{Q}_{rad}$ is the radiative heat transfer in W and m is the slope/gradient of the graph.

With an uncertainty, for $\dot{Q}_{rad}$ of 0.18 mW, if all the $\dot{Q}_{rad}$ values were that much greater, the slope would be $5.74 \cdot 10^{-8}$ and if they were all that much lower, the slope would be $5.42 \cdot 10^{-8}$. Hence, we can conclude that the according to the experimental data calculated, the Stefan-Boltzmann constant lies between $5.42 \cdot 10^{-8} < \sigma < 5.74 \cdot 10^{-8}$, within the bounds of uncertainty of this experiment.

Because the actual Stefan-Boltzmann constant lies in this range, the data validates the radiative law to be true, within the bounds of uncertainty of this experiment.

CLOSURE

The validity of the Stefan-Boltzmann equation (Equation 3) was determined by varying three factors, the diameter of the aperture of the blackbody cavity, the distance between the blackbody cavity and thermopile and the temperature of the blackbody cavity. Based on these three variation of parameters the Stefan-Boltzmann law was tested. For temperature, while keeping other factors constant, a 4^{th} order relation was expected between temperature and radiative heat transfer. Since the blackbody cavity took significant time to heat up, within the time constrains of the lab, only 5 temperature values could be tested. Given this limited data, a 4^{th} degree polynomial fitted to 5 points yields 0 degrees of freedom as it forces the polynomial to pass through every point. Due to this limitation, it could not be positively verified that a 4^{th} order polynomial indeed fit the data. However, the RMS error between expected values and experimental values were low and mostly within uncertainty. Some values significantly varied from expected, due to systematic errors arising from fluctuation of the power meter and very low readings, high uncertainty of power meter at the 30 mW full scale and fluctuating temperature readings at high temperatures.

The diameter between the thermopile and blackbody cavity was then varied, while keeping length and temperature constant. According to the Stefan-Boltzmann law, a 2^{nd} order relation is expected between diameter and radiative heat transfer. Fitting a 2^{nd} order polynomial yielded a p-value of 0.024 and R^2 value of nearly 1. Hence, a second order polynomial fit the data and proved the correlation. Additionally, the error values between expected and theoretical heat transfer was within the uncertainty, except for small aperture/diameters. This was due to the fact that the thermopile was not sensitive to low readings and fluctuated wildly at readings below 0.15 mW.

Finally, the length between the thermopile and blackbody cavity was varied while keeping the temperature and diameter constant. According to the Stefan-Boltzmann law, an inverse proportional 2nd order relation is expected between length and radiative heat transfer. Fitting a $1/x^2$ curve yielded a p-value of 3.86E-13 and R^2 value of 0.998. Hence the data proved the correlation. Additionally, the error values between expected and theoretical heat transfer was within the uncertainty, except for longer lengths. The long distances caused more radiation to be lost to the surroundings and lead to low power meter values.

While the above analysis validated the correlations presented in the Stefan-Boltzmann law, Part 6 of this report validated the accuracy. By plotting all the dependent variables against the radiative heat transfer, the resulting curve should have a linear slope of the Boltzmann constant. From the experimental data, the slope of this graph is between $5.42 \cdot 10^{-8} < \sigma < 5.74 \cdot 10^{-8}$ considering the uncertainty of the radiative heat transfer values. Since the SB constant is indeed within these bounds, the SB law was validated for accuracy within the bounds of uncertainty of this experiment. One major assumption in this experiment was that the emissivity of the blackbody is perfect (equal to 1), which is practically impossible. This lead to slight systematic errors in all the values.

REFERENCES

Bergman, T.L., Lavine, A.S., Incropera, F. P., and DeWitt, D.P., 2011, *Fundamentals of Heat and Mass Transfer*, 7th edn., Wiley, New York.

Coherent Inc., 2006, *FieldMaxII-TO User Manual (Appendix A: Specifications)*, Wilson, Oregon.

Doiron, T., and Beers, J., 1995, “The Gage Block Handbook,” Monograph 180, National Institute of Standards and Technology, Gaithersburg, MD.

H-B Instrument Company, 2009, “Traceable Thermometer Statement (Calibration Certificate #2448.01) against ISO/IEC 17025:2005 and ANSI/NCSL Z540-1-1994,” Collegeville, PA.

Infrared Systems Development, 2013, *Calibration Uncertainty and Traceability Report*, Winter Park, Florida.

Munson, B.R., Young, D.F., Okiishi, T.H., 2002, *Fundamentals of Fluid Mechanics*, John Wiley & Sons, New York, NY.

Attachment 1. Raw data collected to verify SB Law

D1	D2	L (cm)	T1 (K)	T2 (K)	Qrad (mW)	S-theory	F12-theory	Qrad theory (mW)
0.0049	0.011	20	573.15	294.85	0.11	9924.574	0.000507787	0.054486016
0.01	0.011	20	573.15	294.85	0.35	2383.65	0.000507625	0.226858202
0.0151	0.011	20	573.15	294.85	0.722	1045.976	0.000507353	0.516981891
0.02642	0.011	20	573.15	294.85	1.474	342.3458	0.000506355	1.579546654
0.0049	0.011	25	573.15	294.85	0.092	14406.04	0.000349823	0.037536371
0.01	0.011	25	573.15	294.85	0.243	3459.65	0.000349746	0.156302081
0.0151	0.011	25	573.15	294.85	0.501	1517.885	0.000349617	0.356252604
0.02642	0.011	25	573.15	294.85	1.008	496.497	0.000349143	1.089132199
0.0049	0.011	30	573.15	294.85	0.077	19720.49	0.00025555	0.027420738
0.01	0.011	30	573.15	294.85	0.173	4735.65	0.000255509	0.114187169
0.0151	0.011	30	573.15	294.85	0.352	2077.51	0.00025544	0.260287824
0.02642	0.011	30	573.15	294.85	0.75	679.3007	0.000255187	0.796040188
0.0049	0.011	35	573.15	294.85	0.062	25867.93	0.000194819	0.020904279
0.01	0.011	35	573.15	294.85	0.126	6211.65	0.000194795	0.087054238
0.0151	0.011	35	573.15	294.85	0.245	2724.85	0.000194755	0.198451479
0.02642	0.011	35	573.15	294.85	0.515	890.757	0.000194608	0.607068525
0.0049	0.011	40	573.15	294.85	0.072	32848.36	0.000153419	0.016462025
0.01	0.011	40	573.15	294.85	0.112	7887.65	0.000153404	0.068556598
0.0151	0.011	40	573.15	294.85	0.214	3459.905	0.000153379	0.156290538
0.02642	0.011	40	573.15	294.85	0.454	1130.866	0.000153288	0.478173816
0.0049	0.011	45	573.15	294.85	0.031	40661.77	0.000123939	0.013298744
0.01	0.011	45	573.15	294.85	0.086	9763.65	0.000123929	0.055384047
0.0151	0.011	45	573.15	294.85	0.163	4282.676	0.000123913	0.126264612
0.02642	0.011	45	573.15	294.85	0.373	1399.628	0.000123853	0.386353119
0.0049	0.011	50	573.15	294.85	0.05	49308.16	0.000102206	0.010966753
0.01	0.011	50	573.15	294.85	0.08	11839.65	0.000102199	0.04567284
0.0151	0.011	50	573.15	294.85	0.15	5193.163	0.000102188	0.104127386
0.02642	0.011	50	573.15	294.85	0.31	1697.042	0.000102147	0.318642986
0.0049	0.011	20	673.15	294.85	0.139	9924.574	0.000507787	0.107375742
0.01	0.011	20	673.15	294.85	0.609	2383.65	0.000507625	0.447070088
0.0151	0.011	20	673.15	294.85	1.361	1045.976	0.000507353	1.018817647
0.02642	0.011	20	673.15	294.85	2.828	342.3458	0.000506355	3.11281697
0.0049	0.011	25	673.15	294.85	0.083	14406.04	0.000349823	0.073973031
0.01	0.011	25	673.15	294.85	0.389	3459.65	0.000349746	0.308024944
0.0151	0.011	25	673.15	294.85	0.886	1517.885	0.000349617	0.702067994
0.02642	0.011	25	673.15	294.85	1.923	496.497	0.000349143	2.146355844
0.0049	0.011	30	673.15	294.85	0.071	19720.49	0.00025555	0.054038125
0.01	0.011	30	673.15	294.85	0.272	4735.65	0.000255509	0.225028971
0.0151	0.011	30	673.15	294.85	0.629	2077.51	0.00025544	0.512949936
0.02642	0.011	30	673.15	294.85	1.393	679.3007	0.000255187	1.568758605
0.0049	0.011	35	673.15	294.85	0.053	25867.93	0.000194819	0.041196121
0.01	0.011	35	673.15	294.85	0.201	6211.65	0.000194795	0.171558028
0.0151	0.011	35	673.15	294.85	0.462	2724.85	0.000194755	0.391088879
0.02642	0.011	35	673.15	294.85	1.055	890.757	0.000194608	1.196351625
0.0049	0.011	40	673.15	294.85	0.063	32848.36	0.000153419	0.032441758
0.01	0.011	40	673.15	294.85	0.163	7887.65	0.000153404	0.135104678
0.0151	0.011	40	673.15	294.85	0.347	3459.905	0.000153379	0.308002197
0.02642	0.011	40	673.15	294.85	0.816	1130.866	0.000153288	0.942338465
0.0049	0.011	45	673.15	294.85	0.056	40661.77	0.000123939	0.026207872
0.01	0.011	45	673.15	294.85	0.142	9763.65	0.000123929	0.109145494
0.0151	0.011	45	673.15	294.85	0.305	4282.676	0.000123913	0.248830021

D1	D2	L (cm)	T1 (K)	T2 (K)	Qrad (mW)	S-theory	F12-theory	Qrad theory (mW)
0.02642	0.011	45	673.15	294.85	0.628	1399.628	0.000123853	0.761387162
0.0049	0.011	50	673.15	294.85	0.033	49308.16	0.000102206	0.021612211
0.01	0.011	50	673.15	294.85	0.109	11839.65	0.000102199	0.090007593
0.0151	0.011	50	673.15	294.85	0.229	5193.163	0.000102188	0.205204129
0.02642	0.011	50	673.15	294.85	0.528	1697.042	0.000102147	0.627950617
0.0049	0.011	20	773.15	294.85	0.248	9924.574	0.000507787	0.189896439
0.01	0.011	20	773.15	294.85	1.08	2383.65	0.000507625	0.790653608
0.0151	0.011	20	773.15	294.85	2.423	1045.976	0.000507353	1.801802154
0.02642	0.011	20	773.15	294.85	5.57	342.3458	0.000506355	5.505087531
0.0049	0.011	25	773.15	294.85	0.168	14406.04	0.000349823	0.130822986
0.01	0.011	25	773.15	294.85	0.705	3459.65	0.000349746	0.544749111
0.0151	0.011	25	773.15	294.85	1.584	1517.885	0.000349617	1.241623198
0.02642	0.011	25	773.15	294.85	3.98	496.497	0.000349143	3.795879072
0.0049	0.011	30	773.15	294.85	0.098	19720.49	0.00025555	0.095567652
0.01	0.011	30	773.15	294.85	0.474	4735.65	0.000255509	0.397968848
0.0151	0.011	30	773.15	294.85	1.087	2077.51	0.00025544	0.907163615
0.02642	0.011	30	773.15	294.85	2.438	679.3007	0.000255187	2.774385232
0.0049	0.011	35	773.15	294.85	0.057	25867.93	0.000194819	0.072856276
0.01	0.011	35	773.15	294.85	0.331	6211.65	0.000194795	0.30340427
0.0151	0.011	35	773.15	294.85	0.796	2724.85	0.000194755	0.691649567
0.02642	0.011	35	773.15	294.85	1.856	890.757	0.000194608	2.115775028
0.0049	0.011	40	773.15	294.85	0.037	32848.36	0.000153419	0.057373986
0.01	0.011	40	773.15	294.85	0.235	7887.65	0.000153404	0.238935692
0.0151	0.011	40	773.15	294.85	0.592	3459.905	0.000153379	0.544708883
0.02642	0.011	40	773.15	294.85	1.424	1130.866	0.000153288	1.666546985
0.0049	0.011	45	773.15	294.85	0.016	40661.77	0.000123939	0.046349218
0.01	0.011	45	773.15	294.85	0.171	9763.65	0.000123929	0.193026286
0.0151	0.011	45	773.15	294.85	0.448	4282.676	0.000123913	0.440061546
0.02642	0.011	45	773.15	294.85	1.129	1399.628	0.000123853	1.346530494
0.0049	0.011	50	773.15	294.85	0.011	49308.16	0.000102206	0.038221685
0.01	0.011	50	773.15	294.85	0.137	11839.65	0.000102199	0.159180474
0.0151	0.011	50	773.15	294.85	0.354	5193.163	0.000102188	0.362908164
0.02642	0.011	50	773.15	294.85	0.908	1697.042	0.000102147	1.110544932
0.0049	0.011	20	923.15	294.85	0.511	9924.574	0.000507787	0.390204521
0.01	0.011	20	923.15	294.85	2.244	2383.65	0.000507625	1.624657177
0.0151	0.011	20	923.15	294.85	3.69	1045.976	0.000507353	3.702393529
0.02642	0.011	20	923.15	294.85	10.76	342.3458	0.000506355	11.31200804
0.0049	0.011	25	923.15	294.85	0.286	14406.04	0.000349823	0.268818736
0.01	0.011	25	923.15	294.85	1.392	3459.65	0.000349746	1.11936573
0.0151	0.011	25	923.15	294.85	3.81	1517.885	0.000349617	2.55132212
0.02642	0.011	25	923.15	294.85	7.53	496.497	0.000349143	7.799878627
0.0049	0.011	30	923.15	294.85	0.178	19720.49	0.00025555	0.196375087
0.01	0.011	30	923.15	294.85	0.963	4735.65	0.000255509	0.817757534
0.0151	0.011	30	923.15	294.85	2.218	2077.51	0.00025544	1.864065203
0.02642	0.011	30	923.15	294.85	5.62	679.3007	0.000255187	5.700884476
0.0049	0.011	35	923.15	294.85	0.136	25867.93	0.000194819	0.149707117
0.01	0.011	35	923.15	294.85	0.689	6211.65	0.000194795	0.623443591
0.0151	0.011	35	923.15	294.85	1.64	2724.85	0.000194755	1.421220902
0.02642	0.011	35	923.15	294.85	4.43	890.757	0.000194608	4.347553783
0.0049	0.011	40	923.15	294.85	0.087	32848.36	0.000153419	0.117893674
0.01	0.011	40	923.15	294.85	0.529	7887.65	0.000153404	0.490971752

D1	D2	L (cm)	T1 (K)	T2 (K)	Qrad (mW)	S-theory	F12-theory	Qrad theory (mW)
0.0151	0.011	40	923.15	294.85	1.245	3459.905	0.000153379	1.119283067
0.02642	0.011	40	923.15	294.85	2.986	1130.866	0.000153288	3.424467418
0.0049	0.011	45	923.15	294.85	0.071	40661.77	0.000123939	0.095239671
0.01	0.011	45	923.15	294.85	0.419	9763.65	0.000123929	0.396635818
0.0151	0.011	45	923.15	294.85	0.987	4282.676	0.000123913	0.904250789
0.02642	0.011	45	923.15	294.85	2.382	1399.628	0.000123853	2.76688857
0.0049	0.011	50	923.15	294.85	0.047	49308.16	0.000102206	0.07853899
0.01	0.011	50	923.15	294.85	0.329	11839.65	0.000102199	0.327088495
0.0151	0.011	50	923.15	294.85	0.788	5193.163	0.000102188	0.745713859
0.02642	0.011	50	923.15	294.85	1.914	1697.042	0.000102147	2.281978828
0.0049	0.011	20	1073.15	294.85	0.808	9924.574	0.000507787	0.715988887
0.01	0.011	20	1073.15	294.85	3.58	2383.65	0.000507625	2.981094323
0.0151	0.011	20	1073.15	294.85	7.53	1045.976	0.000507353	6.793546655
0.02642	0.011	20	1073.15	294.85	20.18	342.3458	0.000506355	20.75647922
0.0049	0.011	25	1073.15	294.85	0.521	14406.04	0.000349823	0.493257297
0.01	0.011	25	1073.15	294.85	2.489	3459.65	0.000349746	2.053931667
0.0151	0.011	25	1073.15	294.85	5.31	1517.885	0.000349617	4.681438027
0.02642	0.011	25	1073.15	294.85	13.91	496.497	0.000349143	14.31204947
0.0049	0.011	30	1073.15	294.85	0.351	19720.49	0.00025555	0.360329961
0.01	0.011	30	1073.15	294.85	1.791	4735.65	0.000255509	1.50050877
0.0151	0.011	30	1073.15	294.85	3.73	2077.51	0.00025544	3.420385712
0.02642	0.011	30	1073.15	294.85	10.15	679.3007	0.000255187	10.46059106
0.0049	0.011	35	1073.15	294.85	0.266	25867.93	0.000194819	0.27469859
0.01	0.011	35	1073.15	294.85	1.313	6211.65	0.000194795	1.143960816
0.0151	0.011	35	1073.15	294.85	2.61	2724.85	0.000194755	2.607807741
0.02642	0.011	35	1073.15	294.85	7.57	890.757	0.000194608	7.977355522
0.0049	0.011	40	1073.15	294.85	0.236	32848.36	0.000153419	0.216323891
0.01	0.011	40	1073.15	294.85	1.006	7887.65	0.000153404	0.900887352
0.0151	0.011	40	1073.15	294.85	2.339	3459.905	0.000153379	2.053779988
0.02642	0.011	40	1073.15	294.85	6.13	1130.866	0.000153288	6.283578175
0.0049	0.011	45	1073.15	294.85	0.176	40661.77	0.000123939	0.17475591
0.01	0.011	45	1073.15	294.85	0.786	9763.65	0.000123929	0.727789716
0.0151	0.011	45	1073.15	294.85	1.895	4282.676	0.000123913	1.659215822
0.02642	0.011	45	1073.15	294.85	4.98	1399.628	0.000123853	5.076982347
0.0049	0.011	50	1073.15	294.85	0.124	49308.16	0.000102206	0.144111717
0.01	0.011	50	1073.15	294.85	0.633	11839.65	0.000102199	0.600176868
0.0151	0.011	50	1073.15	294.85	1.461	5193.163	0.000102188	1.368315348
0.02642	0.011	50	1073.15	294.85	4.22	1697.042	0.000102147	4.187218218
0.0049	0.011	55	1073.15	294.85	0.077	58787.55	8.57251E-05	0.120873966
0.01	0.011	55	1073.15	294.85	0.509	14115.65	8.57205E-05	0.503404661
0.0151	0.011	55	1073.15	294.85	1.225	6191.364	8.57127E-05	1.147708897
0.02642	0.011	55	1073.15	294.85	2.978	2023.109	8.56842E-05	3.512359196
0.0049	0.011	60	1073.15	294.85	0.069	69099.92	7.29316E-05	0.102834908
0.01	0.011	60	1073.15	294.85	0.405	16591.65	7.29283E-05	0.428280736
0.0151	0.011	60	1073.15	294.85	0.967	7277.282	7.29226E-05	0.97644757
0.02642	0.011	60	1073.15	294.85	2.508	2377.828	7.2902E-05	2.988392707
0.0049	0.011	65	1073.15	294.85	0.055	80245.27	6.2802E-05	0.08855206
0.01	0.011	65	1073.15	294.85	0.323	19267.65	6.27996E-05	0.368798686
0.0151	0.011	65	1073.15	294.85	0.818	8450.914	6.27954E-05	0.840842034
0.02642	0.011	65	1073.15	294.85	2.133	2761.2	6.27801E-05	2.573476619

Attachment 2. Uncertainty for ruler and caliper

Caliper U_B calculation: comparison to gage block

Gage block U_B = 0.2 µm
Gage block rate length = 60 mm
Gage block measured length by caliper = 60.010 mm
Measurement difference = 60-60.011 mm
= 0.01 mm

$$U_{B\_caliper} = \sqrt{U_{B_{gage\,block}}^2 + (measurement\ difference)^2}$$

$$U_{B\_caliper} = \sqrt{(0.2\ \mu m)^2 + (0.01\ mm)^2} = \mathbf{0.01\ mm}$$

Ruler U_B calculation: comparison to caliper

Caliper U_B = 0.01 mm
Arbitrarily chose ruler length = 3 cm
Ruler length measured by caliper = 3.018 cm
Measurement difference = 3-3.018 cm
= 0.018 cm

$$U_{B\_ruler} = \sqrt{U_{B_{caliper}}^2 + (measurement\ difference)^2}$$

$$U_{B\_ruler} = \sqrt{(0.01\ mm)^2 + (0.018\ cm)^2} = \mathbf{0.18\ mm}$$

Attachment 3. Error propagation analysis calculations for *q*

Room T:	23.0	C								
Pressure		kPa								
Constants:	1.1	cm	D_bb1:	2.63	cm	(D1)				
Stefan-Boltzmar	**5.67E-08**	W/(m^2 K^4	D_bb2:	1.004	cm					
			D_sensor:	1.12	cm	(D2)				
T Room (C)	T (K)	L (cm)	D1	D2	S	F12	Q_model (mW)			
21.7	1073.2	20.0	0.490	1.110	6670.02	7.69E-04	**2.6962**			
Effect of room temperature (1.1 deg uncertainty)								Inflence coeff	Uncertainty	Sqaured
21.7	1073.2	20.0	0.490	1.110	6670.02	7.69E-04	2.6962	-8.48E-05	8.48E-05	7.19E-09
22.8	1073.2	20.0	0.490	1.110	6670.02	7.69E-04	2.6961		0.003%	
Effect of BB temperature (0.22 deg uncertainty)								Inflence coeff	Uncertainty	Sqaured
21.7	1073.2	20.0	0.490	1.110	6670.02	7.69E-04	2.6962	8.03E-03	0.00401563	1.61E-05
21.7	1073.4	20.0	0.490	1.110	6670.02	7.69E-04	2.6980		0.149%	
Effect of D1 (0.001 cm uncertainty)								Inflence coeff	Uncertainty	Sqaured
21.7	1073.2	20.0	0.490	1.110	6670.02	7.69E-04	2.6962	1.10E+01	0.110146413	1.21E-02
21.7	1073.2	20.0	0.491	1.110	6642.88	7.69E-04	2.7073		4.085%	
Effect of D2 (0.001 cm uncertainty)								Inflence coeff	Uncertainty	Sqaured
21.7	1073.2	20.0	0.490	1.110	6670.02	7.69E-04	2.6962	4.86E+00	4.86E-02	2.36E-03
21.7	1073.2	20.0	0.490	1.111	6670.03	7.71E-04	2.7011		1.801%	
Effect of L (0.053 cm uncertainty)								Inflence coeff	Uncertainty	Sqaured
21.7	1073.2	20.0	0.490	1.110	6670.02	7.69E-04	2.6962	-2.68E-01	1.34E-01	1.80E-02
21.7	1073.2	20.1	0.490	1.110	6705.39	7.65E-04	2.6820		4.976%	
								Uncertainty of Power (mW) :		**0.18**
										6.69%
It can be seen that the largest uncertainty comes from the length measurements.										
The effect of room temperature is relatively small.										
In your word document or table, you should round the final uncertainty to 2 signicant digits.										

Attachment 4. Guided sample calculation for EPA on one parameter (T_1) of Q

Calculation of model parameters used for uncertainty analysis.

Note: Values of T_1, L and D_1 were chosen to maximize the uncertainty to get an upper bound of the error.

S calculation

$$S = 1 + \frac{4L^2 + D_2^2}{D_1^2}$$

$$S = 1 + \frac{4(0.20)^2 + (1.110)^2}{(0.490)^2} = 6670.02$$

F_{12} calculation

$$F_{12} = \frac{1}{2}\left[S - \sqrt{S^2 - 4\left(\frac{D_2}{D_1}\right)^2}\right]$$

$$F_{12} = \frac{1}{2}\left[6670.02 - \sqrt{6670.02^2 - 4\left(\frac{1.110}{0.490}\right)^2}\right] = 7.69 \cdot 10^{-4}$$

q_{theory} calculation

$$q = A_1 F_{12} \sigma (T_1^4 - T_2^4)$$

$$q = \frac{\pi}{4}(0.490)^2 \cdot 7.69 \cdot 10^{-4} \cdot 5.67 \cdot 10^{-8} \cdot ((1073.2)^4 - (21.7 + 273.15)^4)) = 2.6962 \text{ mW}$$

Sample Calculation for Uncertainty in T_1

1. Calculate the influence coefficient of T_1

$$\frac{\partial q}{\partial T_1} = \frac{\Delta q}{\Delta T_1} = \frac{(q_{exp} - q_{theory})}{((T_1 + U_{c(Table\ 1)}) - T_{1)}}$$

$$\frac{(q_{T_1} - q_{T_1+U_T})}{((T_1 + U_{T_1}) - T_{1)}} = \frac{(2.6962 - 2.6980)}{((1073.2 + 0.22) - 1073.2)} = 8.03 \cdot 10^{-3}\ mW/℃$$

2. Calculate the total uncertainty in T_1

$$U_{q_{T_1}} = \left|\left(\frac{\partial q}{\partial T_1} U_{T_1}\right)\right| = |(8.03 \cdot 10^{-3} \cdot 0.22| = 0.0040\ mW$$

3. Experimental uncertainty relative to theoretical uncertainty percentage calculation for T_1

$$= \frac{U_{q_{T_1}}}{q_{model}} * 100 = \frac{0.0040}{2.6962} * 100 = 0.15\%$$

Attachment 5. Values used for Figures 1,2 for temperature correlation

Diameter Constant							
D1	**D2**	**L (cm)**	**T1 (K)**	**T2 (K)**	**Qrad (mW)**	**Qrad theory (mW)**	**(Qe-Qt)^2**
0.02642	0.011	20	573.15	294.85	1.474	1.579546654	0.01114
0.02642	0.011	20	673.15	294.85	2.828	3.11281697	0.081121
0.02642	0.011	20	773.15	294.85	5.57	5.505087531	0.004214
0.02642	0.011	20	923.15	294.85	10.76	11.31200804	0.304713
0.02642	0.011	20	1073.15	294.85	20.18	20.75647922	0.332328
0.02642	0.011	25	573.15	294.85	1.008	1.089132199	0.006582
0.02642	0.011	25	673.15	294.85	1.923	2.146355844	0.049888
0.02642	0.011	25	773.15	294.85	3.98	3.795879072	0.033901
0.02642	0.011	25	923.15	294.85	7.53	7.799878627	0.072834
0.02642	0.011	25	1073.15	294.85	13.91	14.31204947	0.161644
0.02642	0.011	30	573.15	294.85	0.75	0.796040188	0.00212
0.02642	0.011	30	673.15	294.85	1.393	1.568758605	0.030891
0.02642	0.011	30	773.15	294.85	2.438	2.774385232	0.113155
0.02642	0.011	30	923.15	294.85	5.62	5.700884476	0.006542
0.02642	0.011	30	1073.15	294.85	10.15	10.46059106	0.096467
0.02642	0.011	35	573.15	294.85	0.515	0.607068525	0.008477
0.02642	0.011	35	673.15	294.85	1.055	1.196351625	0.01998
0.02642	0.011	35	773.15	294.85	1.856	2.115775028	0.067483
0.02642	0.011	35	923.15	294.85	4.43	4.347553783	0.006797
0.02642	0.011	35	1073.15	294.85	7.57	7.977355522	0.165939
RMS Error							**0.280733**
Length Constant							
D1	**D2**	**L (cm)**	**T1 (K)**	**T2 (K)**	**Qrad (mW)**	**Qrad theory (mW)**	**(Qe-Qt)^2**
0.02642	0.011	20	573.15	294.85	1.474	1.579546654	0.01114
0.02642	0.011	20	673.15	294.85	2.828	3.11281697	0.081121
0.02642	0.011	20	773.15	294.85	5.57	5.505087531	0.004214
0.02642	0.011	20	923.15	294.85	10.76	11.31200804	0.304713
0.02642	0.011	20	1073.15	294.85	20.18	20.75647922	0.332328
0.0151	0.011	20	573.15	294.85	0.722	0.516981891	0.042032
0.0151	0.011	20	673.15	294.85	1.361	1.018817647	0.117089
0.0151	0.011	20	773.15	294.85	2.423	1.801802154	0.385887
0.0151	0.011	20	923.15	294.85	3.69	3.702393529	0.000154
0.0151	0.011	20	1073.15	294.85	7.53	6.793546655	0.542364
0.01	0.011	20	573.15	294.85	0.35	0.226858202	0.015164
0.01	0.011	20	673.15	294.85	0.609	0.447070088	0.026221
0.01	0.011	20	773.15	294.85	1.08	0.790653608	0.083721
0.01	0.011	20	923.15	294.85	2.244	1.624657177	0.383586
0.01	0.011	20	1073.15	294.85	3.58	2.981094323	0.358688
0.0049	0.011	20	573.15	294.85	0.11	0.054486016	0.003082
0.0049	0.011	20	673.15	294.85	0.139	0.107375742	0.001
0.0049	0.011	20	773.15	294.85	0.248	0.189896439	0.003376
0.0049	0.011	20	923.15	294.85	0.511	0.390204521	0.014592
0.0049	0.011	20	1073.15	294.85	0.808	0.715988887	0.008466
RMS Error							**0.36871**

Attachment 6. Values used for Figures 3,4 for diameter correlation

Temperature Constant							
D1	**D2**	**L (cm)**	**T1 (K)**	**T2 (K)**	**Qrad (mW)**	**Qrad theory (mW)**	**(Qe-Qt)^2**
0.02642	0.011	20	1073.15	294.85	20.18	20.75647922	0.3323283
0.0151	0.011	20	1073.15	294.85	7.53	6.793546655	0.5423635
0.01	0.011	20	1073.15	294.85	3.58	2.981094323	0.358688
0.0049	0.011	20	1073.15	294.85	0.808	0.715988887	0.008466
0.02642	0.011	25	1073.15	294.85	13.91	14.31204947	0.1616438
0.0151	0.011	25	1073.15	294.85	5.31	4.681438027	0.3950902
0.01	0.011	25	1073.15	294.85	2.489	2.053931667	0.1892845
0.0049	0.011	25	1073.15	294.85	0.521	0.493257297	0.0007697
0.02642	0.011	30	1073.15	294.85	10.15	10.46059106	0.0964668
0.0151	0.011	30	1073.15	294.85	3.73	3.420385712	0.095861
0.01	0.011	30	1073.15	294.85	1.791	1.50050877	0.0843852
0.0049	0.011	30	1073.15	294.85	0.351	0.360329961	8.705E-05
0.02642	0.011	35	1073.15	294.85	7.57	7.977355522	0.1659385
0.0151	0.011	35	1073.15	294.85	2.61	2.607807741	4.806E-06
0.01	0.011	35	1073.15	294.85	1.313	1.143960816	0.0285742
0.0049	0.011	35	1073.15	294.85	0.266	0.27469859	7.567E-05
RMS Error							**0.3921118**
Length Constant							
D1	**D2**	**L (cm)**	**T1 (K)**	**T2 (K)**	**Qrad (mW)**	**Qrad theory (mW)**	**(Qe-Qt)^2**
0.02642	0.011	20	573.15	294.85	1.474	1.579546654	328488.31
0.0151	0.011	20	573.15	294.85	0.722	0.516981891	328488.31
0.01	0.011	20	573.15	294.85	0.35	0.226858202	328488.31
0.0049	0.011	20	573.15	294.85	0.11	0.054486016	328488.31
0.02642	0.011	20	673.15	294.85	2.828	3.11281697	453116.11
0.0151	0.011	20	673.15	294.85	1.361	1.018817647	453116.11
0.01	0.011	20	673.15	294.85	0.609	0.447070088	453116.11
0.0049	0.011	20	673.15	294.85	0.139	0.107375742	453116.11
0.02642	0.011	20	773.15	294.85	5.57	5.505087531	597743.91
0.0151	0.011	20	773.15	294.85	2.423	1.801802154	597743.91
0.01	0.011	20	773.15	294.85	1.08	0.790653608	597743.91
0.0049	0.011	20	773.15	294.85	0.248	0.189896439	597743.91
0.02642	0.011	20	923.15	294.85	10.76	11.31200804	852185.61
0.0151	0.011	20	923.15	294.85	3.69	3.702393529	852185.61
0.01	0.011	20	923.15	294.85	2.244	1.624657177	852185.61
0.0049	0.011	20	923.15	294.85	0.511	0.390204521	852185.61
0.02642	0.011	20	1073.15	294.85	20.18	20.75647922	1151627.3
0.0151	0.011	20	1073.15	294.85	7.53	6.793546655	1151627.3
0.01	0.011	20	1073.15	294.85	3.58	2.981094323	1151627.3
0.0049	0.011	20	1073.15	294.85	0.808	0.715988887	1151627.3
RMS Error							919.66859

Attachment 7. Sample Linear Regression Output for Q vs. D^2 relation

SUMMARY OUTPUT								
Regression Statistics								
Multiple R	0.99999836							
R Square	0.99999671							
Adjusted R Square	0.99999014							
Standard Error	0.02688031							
Observations	4							
ANOVA								
	df	*SS*	*MS*	*F*	*Significance F*			
Regression	2	219.8314	109.9157	152121.7	0.001812962			
Residual	1	0.000723	0.000723					
Total	3	219.8322						
	Coefficients	*andard Errc*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*ower 95.0%*	*Upper 95.0%*
Intercept	-0.84172648	0.057894	-14.539	0.043718	-1.577342855	-0.10611	-1.57734	-0.1061101
X Variable 1	230.168083	8.732357	26.35807	0.024141	119.2129622	341.1232	119.213	341.1232
X Variable 2	21407.6604	265.9221	80.5035	0.007908	18028.79954	24786.52	18028.8	24786.521

Attachment 8. Values used for Figures 1,2 for length correlation

Temperature Constant							
D1	D2	L (cm)	T1 (K)	T2 (K)	Qrad (mW	d theory (r	(Qe-Qt)^2
0.02642	0.011	20	1073.15	294.85	20.18	20.75648	0.332328
0.02642	0.011	25	1073.15	294.85	13.91	14.31205	0.161644
0.02642	0.011	30	1073.15	294.85	10.15	10.46059	0.096467
0.02642	0.011	35	1073.15	294.85	7.57	7.977356	0.165939
0.02642	0.011	40	1073.15	294.85	6.13	6.283578	0.023586
0.02642	0.011	45	1073.15	294.85	4.98	5.076982	0.009406
0.02642	0.011	50	1073.15	294.85	4.22	4.187218	0.001075
0.02642	0.011	55	1073.15	294.85	2.978	3.512359	0.28554
0.02642	0.011	60	1073.15	294.85	2.508	2.988393	0.230777
0.02642	0.011	65	1073.15	294.85	2.133	2.573477	0.19402
0.0151	0.011	20	1073.15	294.85	7.53	6.793547	0.542364
0.0151	0.011	25	1073.15	294.85	5.31	4.681438	0.39509
0.0151	0.011	30	1073.15	294.85	3.73	3.420386	0.095861
0.0151	0.011	35	1073.15	294.85	2.61	2.607808	4.81E-06
0.0151	0.011	40	1073.15	294.85	2.339	2.05378	0.08135
0.0151	0.011	45	1073.15	294.85	1.895	1.659216	0.055594
0.0151	0.011	50	1073.15	294.85	1.461	1.368315	0.00859
0.0151	0.011	55	1073.15	294.85	1.225	1.147709	0.005974
0.0151	0.011	60	1073.15	294.85	0.967	0.976448	8.93E-05
0.0151	0.011	65	1073.15	294.85	0.818	0.840842	0.000522
0.01	0.011	20	1073.15	294.85	3.58	2.981094	0.358688
0.01	0.011	25	1073.15	294.85	2.489	2.053932	0.189284
0.01	0.011	30	1073.15	294.85	1.791	1.500509	0.084385
0.01	0.011	35	1073.15	294.85	1.313	1.143961	0.028574
0.01	0.011	40	1073.15	294.85	1.006	0.900887	0.011049
0.01	0.011	45	1073.15	294.85	0.786	0.72779	0.003388
0.01	0.011	50	1073.15	294.85	0.633	0.600177	0.001077
0.01	0.011	55	1073.15	294.85	0.509	0.503405	3.13E-05
0.01	0.011	60	1073.15	294.85	0.405	0.428281	0.000542
0.01	0.011	65	1073.15	294.85	0.323	0.368799	0.002098
0.0049	0.011	20	1073.15	294.85	0.808	0.715989	0.008466
0.0049	0.011	25	1073.15	294.85	0.521	0.493257	0.00077
0.0049	0.011	30	1073.15	294.85	0.351	0.36033	8.7E-05
0.0049	0.011	35	1073.15	294.85	0.266	0.274699	7.57E-05
0.0049	0.011	40	1073.15	294.85	0.236	0.216324	0.000387
0.0049	0.011	45	1073.15	294.85	0.176	0.174756	1.55E-06
0.0049	0.011	50	1073.15	294.85	0.124	0.144112	0.000404
0.0049	0.011	55	1073.15	294.85	0.077	0.120874	0.001925
0.0049	0.011	60	1073.15	294.85	0.069	0.102835	0.001145
0.0049	0.011	65	1073.15	294.85	0.055	0.088552	0.001126
RMS Error							0.290677

Diameter Constant							
D1	**D2**	**L (cm)**	**T1 (K)**	**T2 (K)**	**qnet (mW**	**Q suppose**	**(Qe-Qt)^2**
0.02642	0.011	20	573.15	294.85	1.474	1.579547	0.01114
0.02642	0.011	25	573.15	294.85	1.008	1.089132	0.006582
0.02642	0.011	30	573.15	294.85	0.75	0.79604	0.00212
0.02642	0.011	35	573.15	294.85	0.515	0.607069	0.008477
0.02642	0.011	40	573.15	294.85	0.454	0.478174	0.000584
0.02642	0.011	45	573.15	294.85	0.373	0.386353	0.000178
0.02642	0.011	50	573.15	294.85	0.31	0.318643	7.47E-05
0.02642	0.011	20	673.15	294.85	2.828	3.112817	0.081121
0.02642	0.011	25	673.15	294.85	1.923	2.146356	0.049888
0.02642	0.011	30	673.15	294.85	1.393	1.568759	0.030891
0.02642	0.011	35	673.15	294.85	1.055	1.196352	0.01998
0.02642	0.011	40	673.15	294.85	0.816	0.942338	0.015961
0.02642	0.011	45	673.15	294.85	0.628	0.761387	0.017792
0.02642	0.011	50	673.15	294.85	0.528	0.627951	0.00999
0.02642	0.011	20	773.15	294.85	5.57	5.505088	0.004214
0.02642	0.011	25	773.15	294.85	3.98	3.795879	0.033901
0.02642	0.011	30	773.15	294.85	2.438	2.774385	0.113155
0.02642	0.011	35	773.15	294.85	1.856	2.115775	0.067483
0.02642	0.011	40	773.15	294.85	1.424	1.666547	0.058829
0.02642	0.011	45	773.15	294.85	1.129	1.34653	0.04732
0.02642	0.011	50	773.15	294.85	0.908	1.110545	0.041024
0.02642	0.011	20	923.15	294.85	10.76	11.31201	0.304713
0.02642	0.011	25	923.15	294.85	7.53	7.799879	0.072834
0.02642	0.011	30	923.15	294.85	5.62	5.700884	0.006542
0.02642	0.011	35	923.15	294.85	4.43	4.347554	0.006797
0.02642	0.011	40	923.15	294.85	2.986	3.424467	0.192254
0.02642	0.011	45	923.15	294.85	2.382	2.766889	0.148139
0.02642	0.011	50	923.15	294.85	1.914	2.281979	0.135408
0.02642	0.011	20	1073.15	294.85	20.18	20.75648	0.332328
0.02642	0.011	25	1073.15	294.85	13.91	14.31205	0.161644
0.02642	0.011	30	1073.15	294.85	10.15	10.46059	0.096467
0.02642	0.011	35	1073.15	294.85	7.57	7.977356	0.165939
0.02642	0.011	40	1073.15	294.85	6.13	6.283578	0.023586
0.02642	0.011	45	1073.15	294.85	4.98	5.076982	0.009406
0.02642	0.011	50	1073.15	294.85	4.22	4.187218	0.001075
0.02642	0.011	55	1073.15	294.85	2.978	3.512359	0.28554
0.02642	0.011	60	1073.15	294.85	2.508	2.988393	0.230777
0.02642	0.011	65	1073.15	294.85	2.133	2.573477	0.19402
RMS Error							**0.280421**

Attachment 9. Sample Linear Regression Output for Q vs. L^{-2} relation

SUMMARY OUTPUT								
Regression Statistics								
Multiple R	0.999455							
R Square	0.99891							
Adjusted R Square	0.998774							
Standard Error	0.202863							
Observations	10							
ANOVA								
	df	*SS*	*MS*	*F*	*Significance F*			
Regression	1	301.7218	301.7218	7331.602	3.86116E-13			
Residual	8	0.329229	0.041154					
Total	9	302.051						
	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
Intercept	-0.23094	0.110529	-2.08941	0.070082	-0.485820791	0.02394	-0.4858208	0.0239395
X Variable 1	1.220804	0.014258	85.62478	3.86E-13	1.187925481	1.253682	1.1879255	1.2536816
		Here the X Variable 1 is x^(-2)						

Attachment 10. Linear Regression Output for Q vs. $\varepsilon A_{s_1} F_{12}(T_1^4 - T_2^4)$ (Part 6 of Analysis)

SUMMARY OUTPUT								
Regression Statistics								
Multiple R	0.99720588							
R Square	0.99441956							
Adjusted R Square	0.98779705							
Standard Error	0.00022733							
Observations	152							
ANOVA								
	df	*SS*	*MS*	*F*	*Significance F*			
Regression	1	0.001391	0.001391	26907.81	4E-171			
Residual	151	7.8E-06	5.17E-08					
Total	152	0.001398						
	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
X Variable 1	5.5796E-08	3.4E-10	164.036	4.9E-172	5.51E-08	5.65E-08	5.51E-08	5.65E-08