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Undergraduate Instructional Laboratories

TO: Michael Baldwin — June 6, 2017

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SUBJECT: Lab 2: Forced Convection

INTRODUCTION

The forced convection experiment was performed at the Georgia Institute of Technology Mechanical Engineering Thermal Laboratory at the George W. Woodruff School of Mechanical Engineering on May 30, 2017. The objective of the lab was to determine the convection coefficient for a series of air flow velocities onto a copper cylinder by observing the ambient temperature of the wind tunnel and the surface temperature of the copper cylinder with thermocouples under the range of air velocities. The convection coefficient result was compared to literature and developed into a model to determine the convection coefficient empirically. The uncertainties of the experimental convection coefficient and the empirical model were calculated. Essentially, the relationship between convection coefficient and velocity fluctuation was investigated.

APPARATUS AND UNCERTAINTY

Apparatus. The data collected in this experiment required the use of two separate thermocouple arrangements. The Omega K-Type thermocouples were used to measure the temperatures within the Gunt Hamburg HM170 Educational Wind Tunnel. The ambient temperature was measured with a single thermocouple centered near the mouth of the tunnel. The surface temperature of the copper cylinder, heated by a Weston Instruments Model 310 0 to 125 W with 1 W marking wattmeter, was measured by 14 thermocouples mounted to its surface. The air velocity within the wind tunnel was measured by a thermal anemometer, specifically a TSI Velocicalc Model 8350; S/N: 291. For data calculations and analysis, the diameter of the copper cylinder was measured with a Fowler ST133 0 to 20 cm calipers with 0.02 mm marking and the length was measured with a ruler. Gage blocks, specifically Weber Gage Dev. – Starett Model RS45MA1 Grade 2, were used to determine the uncertainty of the calipers, in turn determining the uncertainty of the ruler. To measure the ambient conditions a thermometer was used to measure the temperature, specifically a VWR General Purpose Glass Thermometer, Cat. #89095-598; - 20 °C to 110 °C with 1 °C markings, to measure the pressure a barometer was used, specifically O-N Ins. Aneroid Barometer; 500-780 mmHg with 5 mmHg markings. Table 1 lists the equipment used in this experiment and their associated uncertainties. The references for the uncertainty values are displayed below the table.

Table 1. Uncertainty of all utilized measurement devices.

Generic ID	Commercial ID	U_A	U_B	U_C
Gage Blocks	Weber Gage Div. – Starett Model RS45MA1, Grade 2	N/A	0.2 μm[6]	0.2 μm
Caliper	Fowler ST133 0 to 20 cm w/ 0.02 mm marking	0.018 mm[7]	0.01 mm[8]	0.021 mm
Ruler	-	0.22 mm[7]	4.4 mm[8]	4.4 mm
Thermocouple (surface)	Omega K-Type	1.1°C[7]	1.5°C[3]	1.9°C
Thermocouple (air)	Omega K-Type	0.11°C[7]	1.5°C[3]	1.5°C
Wattmeter	Weston Instruments Model 310. 0 to 125 W w/ 1 W marking	0.5 W[1]	0.078 W[2]	0.51 W
Thermal Anemometer	TSI Velocialc Model 8350; S/N: 291	0.20 m/s[1]	0.46 m/s[4]	0.50 m/s
Thermometer	VWR General Purpose Glass Thermometer, Cat. #89095-598; -20 °C to 110 °C w/ 1 °C markings	0.5 °C [1]	1 °C [5]	1.1 °C
Barometer	O-N Ins. Aneroid Barometer; 500-780 mmHg with 5 mmHg markings	330 Pa [1]	N/A	330 Pa

[1] By inspection; [2] Weston (1974); [3] Omega Engineering Inc, (2008); [4] TSI, Inc. (1996); [5] H-B Instrument Company (2009). [6] Doirion T. and Beers (1995); [7] By method of convergence of standard deviation; [8] By comparison

Uncertainty.

Three types of uncertainty are found for each apparatus. Type A uncertainty, U_A, is the uncertainty associated with error by the user. Type B uncertainty, U_B, is uncertainty associated with the device. Type C uncertainty, U_C, is found by relating Type A and Type B uncertainties, using Equation 1.

$$U_c = \sqrt{U_A^2 + U_B^2} \qquad (1)$$

Type A Uncertainty.

The U_A of the wattmeter, thermometer, and barometer is 0.5 W, 0.5 °C, and 330 Pa respectively, determined by taking half of the smallest graduation. The U_A of the thermal anemometer is 0.20 m/s as determined by taking the maximum fluctuation between the readings at one fan speed. The U_A of the caliper, ruler, and thermocouples (surface and air) is 0.018 mm, 0.22 mm, 1.1°, and

0.11°C respectively, determined by the mean U_A based upon standard deviation because each of these devices took multiple measurements without changing any experimental variables.

For example, the length and the diameter of the copper cylinder was measured 50 times each with the caliper and ruler. The average of the 50 measurements is averaged and used as the diameter and length dimensions in the data analysis calculations. This mean U_A is calculated by Equation 2,

$$U_A = \frac{k_c SSD}{\sqrt{N}} \tag{2}$$

where k_c is the coverage factor, *SSD* is the student's standard deviation, and *N* is the number of data points taken.

The coverage factor can be calculated in excel using the call function TINV(probability, degree of freedom), where the probability is 0.05 to achieve a 95% confidence interval and the degree of freedom is *N*-1. The *SSD* can be calculated with Equation 3,

$$SSD = \sqrt{\frac{\sum_i (x_i - \bar{x})^2}{N-1}} \tag{3}$$

Where x_i is a data point and $\bar{x}$ is the mean of all *N* data points. The *N* for the caliper and ruler is 50 because the diameter and length was measured 50 times each and then averaged for the diameter and length values used in analysis calculations. The *N* for the surface thermocouples is 14 because the copper cylinder has 14 thermocouples mounted to the surface which each read a temperature and record the average of the 14 readings. The *N* for the air thermocouple is 18 because the ambient temperature was measured at each fan speed two different times, so the average of these 18 temperatures is the ambient temperature used in data analysis calculations.

The U_A uncertainties are calculated using these equations and are shown in detail in Attachment 1.

<u>Type B Uncertainty.</u>

The U_B of the gage blocks, thermocouples (surface and air), and thermometer is 0.2 µm, 1.5°C, and 1°C respectively, determined by the uncertainty data provided by the respective manufacturers: Doirion T. and Beers (1995), Omega Engineering Inc. (2008), and H-B Instruments Company (2009). The U_B of the wattmeter is 0.078 W, determined by taking 0.25% of the wattmeter's full scale of 125 W which is specified by the device's manufacturer: Weston (1974). The U_B of the thermal anemometer is 0.46 m/s, determined by taking 3% of the maximum velocity reading which is specified by the device's manufacturer: TSI Inc. (1996). The U_B of the caliper and ruler is 0.01 mm and 0.044 cm respectively, calculated with Equation 4,

$$U_B = \sqrt{U_{B_{\mathrm{ref}}}^2 + (\text{measurement difference})^2} \qquad (4)$$

where U_{Bref} is the type B uncertainty for the device being compared to the device in which the type B uncertainty is unknown and the measurement difference is the difference between the rated or listed measurement and the actual measurement confirmed by the secondary device. Please see the detailed caliper and ruler U_B calculations in Attachment 4 for clarification.

PROCEDURE

The following procedure was used to complete the experiment. Using these steps, several data points were recorded and calculations were made as discussed in the following section.

1. Record ambient room temperature and pressure readings.

2. Measure the length and diameter dimensions of the copper cylinder.

3. Turn on the wind tunnel and set the fan speed to 4.6, then wait 5-10 minutes for the wind tunnel to reach steady state.

4. Set the power supply to 50 W.

5. Measure and record the wind speed with a thermal anemometer.

6. Measure and record the ambient temperature of the wind tunnel with a single thermocouple and the average surface temperature of the copper cylinder and the temperature's standard deviation with 14 thermocouples mounted to the cylinder.

7. Decrease the fan speed in intervals of 0.5 until the final fan speed of 0.6 and record the wind speed, ambient temperature, average surface temperature of the copper cylinder, and the temperature standard deviation at each fan speed interval. Wait 5 minutes at each fan speed interval for the wind tunnel to reach steady state before taking any temperature measurements.

DATA ANALYSIS AND FINDING

<u>Part 1. Calculation of h, the heat transfer coefficient</u>

Based on an energy balance done on the system consisting of the copper rod, and the surrounding of everything outside the surface of the copper rod, the following equation was derived,

$$50\,W = \dot{Q}_{conv} + \dot{Q}_{Leak} \tag{5}$$

where 50 W is the input power from the wattmeter, $\dot{Q}_{conv}$ refers to Newton's second law of cooling, for loss of heat from the copper surface to the surrounding via convection, and $\dot{Q}_{Leak}$ is the energy loss due to radiation to the surrounding, and conduction within the copper rod.

The $\dot{Q}_{Leak}$ was further calculated using the Stefan-Boltzmann Law of Radiation and the Power Law for conduction, as shown in Equation 6,

$$\dot{Q}_{Leak} = \sigma\varepsilon_0 A(T_s^4 - T_\infty^4)\cdot F_{12} + kA(\,T_s - \,T_\infty) \tag{6}$$

where $\sigma = 5.67\cdot 10^{-8}\,\frac{\mathrm{W}}{\mathrm{m^2K^4}}$ is the Stefan-Boltzmann constant, $\varepsilon_0 = 0.3$ is the emissivity of the surface, $A = 0.021427\ \mathrm{m}^2$ is the area of the cylinder, T_s is the temperature of the copper surface in Kelvin, T_∞ is the steady-state surrounding temperature in Kelvin, $F_{12} = 1$ is the view factor of the copper rod's geometry, and k is the thermal conductivity that is a linear function of temperature.

As seen in Attachment 1, this is calculated in the $\dot{Q}_{Leak}$ column.

Using Newton's Law of cooling as shown in Equation 7,

$$\dot{Q}_{conv} = Ah(\,T_s - \,T_\infty) \tag{7}$$

where $A = 0.021427\ \mathrm{m}^2$ is the area of the cylinder, h is the heat convection coefficient, T_s is the temperature of the copper surface in Kelvin, and T_∞ is the steady-state surrounding temperature in Kelvin.

Hence, the heat transfer coefficient can be calculated as seen in Attachment 1, using Equation 8,

$$h = \frac{50 - \dot{Q}_{Leak}}{A(T_s - T_\infty)} \qquad (8)$$

where $\dot{Q}_{Leak}$ is calculated from Equation 6, $A = 0.021427$ m^2 is the area of the cylinder, T_s is the temperature of the copper surface in Kelvin, and T_∞ is the steady-state surrounding temperature in Kelvin.

Part 2. Error Analysis or h, the heat transfer coefficient

The surface area of the cylinder is calculated with Equation 9,

$$A = \pi DL \qquad (9)$$

where $D = 0.031767$ m is the diameter of the copper rod and $L = 0.2147$ m is the length of the copper rod.

Performing an EPA on the surface area of the cylinder results in Equation 10,

$$U_A = \sqrt{\left(\frac{\partial A}{\partial L}U_L\right)^2 + \left(\frac{\partial A}{\partial D}U_D\right)^2} = \sqrt{(\pi DU_L)^2 + (\pi LU_D)^2} \qquad (10)$$

where U_A is the total uncertainty in the surface area of the copper rod, $L = 0.2147$ m is the length of the rod, $U_L = 0.0044$ m is the total uncertainty of the length of the rod, $D = 0.03177$ m is the diameter of the rod and $U_D = 0.0205$ mm is the uncertainty of the diameter of the rod.

With the values above, the uncertainty of the surface area was found to be $U_A = 0.00044$ m^2. The uncertainty of the heat transfer coefficient can be calculated from Equation 11. Using that equation, an error propagation analysis was carried out as shown in Equation 11,

$$U_h = \sqrt{\left(\frac{\partial h}{\partial T_\infty}U_{T\infty}\right)^2 + \left(\frac{\partial h}{\partial T_s}U_{T_s}\right)^2 + \left(\frac{\partial h}{\partial Q_E}U_{Q_E}\right)^2 + \left(\frac{\partial h}{\partial Q_{Leak}}U_{Q_{Leak}}\right)^2 + \left(\frac{\partial h}{\partial \mathrm{A}}U_{\mathrm{A}}\right)^2} \qquad (11)$$

where U_h is the total combined uncertainty for the heat transfer coefficient, $\dot{Q}_{Leak}$ is calculated from Equation 6, $U_{Q_{Leak}} = 0.06\,W$ as referenced by Pascual and Jeter (1998), $A = 0.021427$ m^2 and $U_{\mathrm{A}} = 0.00044$ m^2 is the area and uncertainty of area of the cylinder, T_s and $U_{T_s} = 1.84$°C is the temperature and uncertainty of temperature of the copper surface, T_∞ and $U_{T_\infty} = 1.504$°C is the steady-state surrounding temperature and uncertainty, $Q_E = 50\text{W}$ and $U_{Q_E} = 0.59\text{W}$ is the electric power and uncertainty of power.

$$U_C = \sqrt{\left(\frac{\dot{Q}_E - \dot{Q}_{Leak}}{A_s(\mathrm{T}_s - T_\infty)^2} U_{T_\infty}\right)^2 + \left(\frac{\dot{Q}_{Leak} - \dot{Q}_E}{A_s(\mathrm{T}_s - T_\infty)^2} U_{\mathrm{T}_s}\right)^2 + \left(\frac{1}{A_s(\mathrm{T}_s - T_\infty)} U_{Q_E}\right)^2 + \left(\frac{-1}{A_s(\mathrm{T}_s - T_\infty)} U_{Q_{Leak}}\right)^2 + \left(\frac{\dot{Q}_{Leak} - \dot{Q}_E}{{A_s}^2(\mathrm{T}_s - T_\infty)} U_{A_s}\right)^2} \quad (12)$$

Using the values above, the uncertainty was computed as shown in Attachment 1, as 10.8 $\frac{\text{W}}{\text{m}^2\text{C}}$. The values used for T_∞, T_s and Q_{Leak} were chosen from the data such that uncertainty is maximized and the uncertainty presents an upper bound limit for the error.

If typical values (average) were used for T_∞, T_s and Q_{Leak}, then the uncertainty is 5.97≈ 6 $\frac{\text{W}}{\text{m}^2\text{C}}$.

Part 3. Analysis of varying convection coefficient h with velocity V

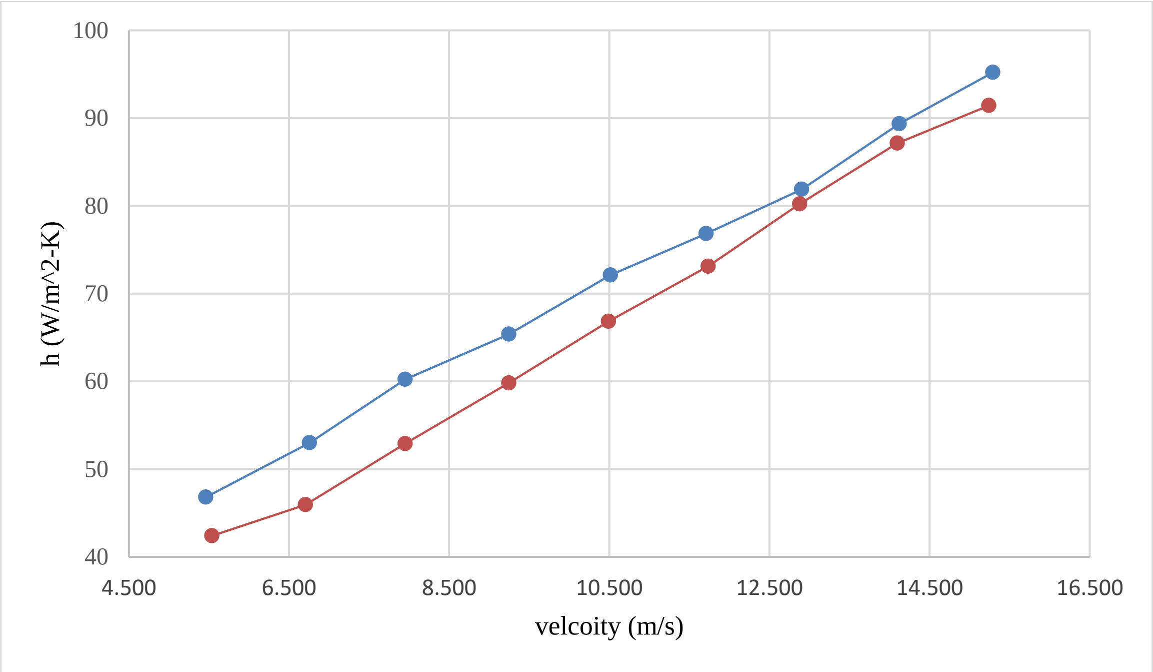

Figure 1. Plot of calculated convection coefficient against velocity for both sets of data

As noted in Figure 1, the two sets of data were consistently more than or less than the other set of data. For the dataset that's greater (blue line), the experiment started with the greatest fan speed (highest velocity), and for subsequent values, the velocity was steadily increased. However, for each reading, only 5 minutes was allotted to allow the copper rod to reach its steady-state temperature which was not quite enough. Hence, since velocities were decreasing, and the copper rod was heating, not enough time was given for it to heat. So, the surface temperatures recorded are lower than expected. As seen in Equation 8 for the convection transfer coefficient, it is inversely proportional to the surface temperature. Hence, since the surface temperatures recorded are lower than actual values, the convection coefficient is larger than the actual values.

Another set of data was produced by starting at the lowest fan speed setting (velocity) and subsequently increasing fan speed for further readings. Here, the opposite occurs where the sample is not allowed ample time to let the copper rod cool. So surface temperatures recorded are higher than steady-state values, which makes the convection coefficient constantly lower than expected.

Hence, it can be concluded that realistic convection coefficient values lie between the two curves.

The largest difference that occurred between the two datasets was for $V \approx 8\,\mathrm{m/s}$ where the difference in h between the two data sets is $7.3\,\frac{\mathrm{W}}{\mathrm{m^2C}}$ which is still within the uncertainty of the convection coefficient, but is not within the typical value uncertainty of h. However, the difference between the two datasets can be improved/reduced by allowing more time for the copper rod to reach stead-state.

To achieve better results, a longer time is required between trials to allow the copper rod to reach its steady-state temperature for each velocity value. The amount of time required can be computed from the thermal time constant in Equation 13,

$$\tau = \frac{1}{hA}\rho V c \qquad (13)$$

where h is the heat transfer coefficient, $A = 0.0214\,\mathrm{m}^2$ is the surface area of the cylinder, $\rho = 8960\,\frac{\mathrm{kg}}{\mathrm{m}^3}$ is the density of the copper rod, $V = 1.7 \cdot 10^{-4}\,\mathrm{m}^3$ is the volume of the copper rod and $c = 385\,\frac{\mathrm{J}}{\mathrm{kgK}}$ is the thermal conductivity of the copper rod.

The 99.7% value, or 3 time constants, is considered close enough that steady-state is considered to be reached.

Based on the lowest value of $h = 42.397 \frac{\text{W}}{\text{m}^2\text{C}}$ reached in the experiment, $3\tau \approx 32$ minutes and based on the highest value of $h = 95.203 \frac{\text{W}}{\text{m}^2\text{C}}$ reached in the experiment, $3\tau \approx 14.5$ minutes. So, at worst, the time between data recording should have been at least 14.5 minutes but more practically, since convection coefficient vales reached 42.397, the time to allow steady-state to be reached is closer to 30 minutes.

Since we allowed only 5 minutes between each trail, more than 6 times less than the 32 minutes, it can be concluded that the difference between the measured h between the two datasets is primarily due bias uncertainty.

Part 4. Comparison with Literature

After solving for the experimental convection coefficient through forced convection, it is important to check this value with what the theoretical value should be using equations found in literature. In order to do so, the experimental Nusselt Number will be solved for as a function of the convection coefficient, and the theoretical Nusselt numbers will be compared to each experimental Nusselt Number at each corresponding velocity. Before solving for the dimensionless properties of the fluid, such as Nusselt Number, Reynolds Number, and Prandtl Number, the other relevant fluid properties must be determined first. Dynamic viscosity, thermal conductivity, and specific heat can all be solved by fitting a quadratic equation to those values versus the temperature. These quadratic fits are indicated in Equations 14, 15, and 16 respectively,

$$\mu(T) = 1.076 \cdot 10^{-6} + 6.705 * 10^{-8} \cdot T - 3.043 \cdot 10^{-11} \cdot T^2 \qquad (14)$$

where μ is dynamic viscosity in $\frac{\text{N}\cdot\text{s}}{\text{m}^{\text{s}}}$ and T is temperature in Kelvin.

$$k(T) = 8.57 \cdot 10^{-5} + 9.624 \cdot 10^{-5} \cdot T - 3 \cdot 10^{-8} \cdot T^2 \qquad (15)$$

where k is thermal conductivity in $\frac{\text{W}}{\text{m}\cdot\text{K}}$ and T is temperature in Kelvin.

$$c_p(T) = 1033 - 0.211 \cdot T + 4.1 \cdot 10^{-4} \cdot T^2 \qquad (16)$$

where c_p is specific heat in $\frac{\text{J}}{\text{kg}\cdot\text{K}}$ and T is temperature in Kelvin.

The temperatures used for solving the equations above are the temperature of the air inside the wind tunnel at each given velocity, and they were assumed to be the average of the temperature near the surface of the heated cylinder and the temperature from far away. This is shown in Equation 17,

$$T_{film} = 273.15 + \frac{T_\infty + T_s}{2} \qquad (17)$$

where T_{film} is Temperature of the film in Kelvins, T_∞ is the steady-state surrounding temperature in Celsius, and T_s is the surface temperature of copper rod in Celsius.

With all the fluid properties and geometric properties determined, the dimensionless parameters can be equated, and these equations are shown in Equation 18, 19, and 20,

$$Pr = \frac{\mu C_p}{k} \qquad (18)$$

where Pr is Prandtl Number, μ is dynamic viscosity in $\frac{\mathrm{N \cdot s}}{\mathrm{m^s}}$, c_p is specific heat in $\frac{\mathrm{J}}{\mathrm{kg \cdot K}}$, and k is thermal conductivity in $\frac{\mathrm{W}}{\mathrm{m \cdot K}}$.

$$Re = \frac{\rho v D}{\mu} \qquad (19)$$

where Re is Reynolds Number, ρ is density in $\frac{\mathrm{kg}}{\mathrm{m^3}}$, v is velocity in $\frac{\mathrm{m}}{\mathrm{s}}$, $D = 0.0317676$ m is the diameter of the of the copper cylinder, and μ is dynamic viscosity in $\frac{\mathrm{N \cdot s}}{\mathrm{m^s}}$.

$$Nu = \frac{hD}{k} \qquad (20)$$

where Nu is the experimental Nusselt Number, h is the convection coefficient in $\frac{\mathrm{W}}{\mathrm{m^2 K}}$, $D = 0.0317676$ m is the diameter of the of the copper cylinder, and k is thermal conductivity in $\frac{\mathrm{W}}{\mathrm{m \cdot K}}$.

The Prandtl Number shown above is considered the ratio of the viscous diffusion to the thermal diffusion. For most gases, the Prandtl Number remains constant at any given temperature and is a number that is only a function of the type of fluid. This is shown in Attachment 2 because the Prandtl number remains almost the same despite the varying velocity and temperature. The Reynolds Number shown above is the ratio of the inertial force to the viscous force. It is used to classify the fluid flow as either laminar or turbulent. This critical Reynolds Number will depend on the environment in which it flows in. This experiment was conducted in external flow, so the critical Reynolds number is around 500,000.

For this experiment, the maximum Reynolds number shown in Attachment 2 is about 30,000, which is far below the critical number; thus, the flow is in the laminar regime. The final dimensionless parameter shown above is the Nusselt Number. This is defined as the ratio of the total heat transfer to the conduction heat transfer. Therefore, a large Nusselt Number indicates that there is efficient convection.

Determining the theoretical Nusselt Numbers was done by solving three equations from literature. These equations are the Fand & Keswani, Hilpert, and Churchill & Bernstein equations shown in Equation 21, 22, and 23 respectively,

$$Nu_1 = 0.184 + 0.324\sqrt{Re} + 0.291Re^{0.247+0.0407Re^{0.168}} \qquad (21)$$

$$Nu_2 = 0.193Re^m Pr^{1/3} \qquad (22)$$

$$Nu_3 = 0.3 + \frac{0.62Re^{1/2}Pr^{1/3}}{[1 + (\frac{0.4}{Pr})^{\frac{2}{3}}]^{1/4}}[1 + (\frac{Re}{282000})^{5/8}]^{4/5} \qquad (23)$$

where Nu_1 is the Nusselt Number of the Fand & Keswani equation, Nu_2 is the Nusselt Number of the Hilpert equation, Nu_3 is the Nusselt Number of the Churchill & Bernstein equation, Re is the Reynolds number, and the m in Equation 22 is 0.618.

Each equation determines the Nusselt Number as a function of both the Reynolds Number and the Prandtl Number. The standard deviations of the difference between the experimental and theoretical Nusselt Numbers are shown in Attachment 2. These equations show a max standard deviation of about 11.79% of the average experimental Nusselt Number. A visual representation of the data can be seen in Figure 2.

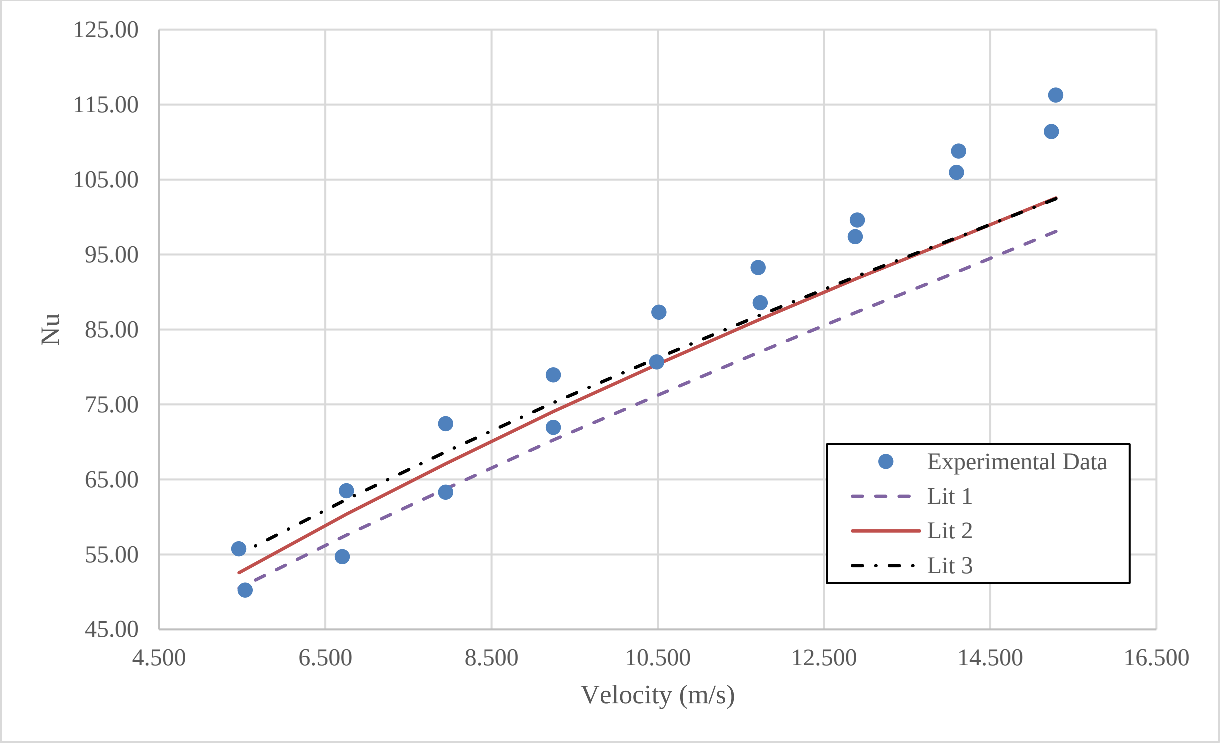

Figure 2. Plot of Experimental and Literature Nusselt Number versus velocity.

From the figure, literature value 1 varies much more from the experimental data. This lines up with the results shown in Attachment 2, which shows that literature value 1 had the highest standard deviation of the difference from the experimental data. The figure also shows that as the velocity increases, the theoretical and experimental Nusselt numbers seem to vary much more. At high velocities, the Nusselt number's experimental value is as much as 18 greater than the theoretical value. In order to see if these numbers are within the uncertainty of Nu, an EPA must be performed on Nu. From Equation 20, the EPA of Nu can be performed by using Equation 24,

$$U_{Nu} = \sqrt{\left(\frac{\partial Nu}{\partial \mathrm{h}} U_{\mathrm{h}}\right)^2 + \left(\frac{\partial Nu}{\partial \mathrm{D}} U_{\mathrm{D}}\right)^2} \tag{24}$$

where U_{Nu} is the total combined uncertainty for the Nusselt Number, $h = 95.203 \frac{\mathrm{W}}{\mathrm{m^2C}}$ is the convection coefficient (based on the largest value of h), $U_{\mathrm{h}} = 10.8 \ \frac{\mathrm{W}}{\mathrm{m^2C}}$ when maximized and $U_{\mathrm{h}} = 5.97 \ \frac{\mathrm{W}}{\mathrm{m^2C}}$ when typical values are used to calculate it, $D = 0.03177$ m is the diameter of the rod and $U_D = 0.0205$ mm is the uncertainty of the diameter of the rod, and $k = 0.0263 \ \frac{\mathrm{W}}{m\mathrm{K}}$ is assumed to be the constant thermal conductivity.

Equation 24 can be rewritten below as Equation 25,

$$U_{Nu} = \sqrt{\left(\frac{\mathrm{D}}{\mathrm{k}} U_{\mathrm{h}}\right)^2 + \left(\frac{\mathrm{h}}{\mathrm{k}} U_{\mathrm{D}}\right)^2} \tag{25}$$

Using the values above, the uncertainty of the Nusselt Number was computed to be 13.04 when U_{h} is maximized and 7.212 when typical values are used to calculate U_{h}.

The standard deviation of Literature 2 and 3 is around 6.60. Hence, 2 standard deviations (95% confidence) can fit within the maximum uncertainty of the Nusselt Number calculated. Hence, these two models can be considered valid within the uncertainty of the experiment. However, the standard deviation of Literature 1 is 9.82, which is almost identical to the max uncertainty. This refers to about 68% confidence interval (1 standard deviation) and hence, the model cannot be considered to fit the experimental data.

Because the uncertainty of the Nusselt Number is less than the actual error that was found, the data that was found experimentally does not line up with the theoretical data. There are numerous explanations for this outcome. A high Nusselt Number corresponds to a high convection coefficient meaning that the surface temperature was likely measured too low. An explanation for this is that the experiment was not conducted with enough time to heat up/cool down and reach steady state. To reach steady-state, it takes around three time constants. Given that the settling time was much greater than the 5 minutes given to heat up/cool down (as shown in part 3), the experiment was still in the transient state when the data was taken. Another explanation is that the Q_{Leak} was underestimated. If more heat were lost to conduction and radiation than was expected, this could account for the large Nusselt Number values. This could be due to incorrect assumptions of the emissivity, view factor, or thermal conductivity constants.

Another cause of the different results was that our film temperature could have been poorly assumed from Equation 17. It was assumed to be the average of the surface temperature and a temperature from far away from the heater. It is possible that the true temperature to be used should have been some other weighted average of the two temperatures. Lastly, assuming the thermal conductivity to be constant in Equation 24 and 25 above could mess up the results. Thermal conductivity should, in reality, fluctuate with temperature; thus, slightly incorrect uncertainties may have been attained for the Nusselt Number.

Part 5. Regression Analysis of Reynolds and Nusselt Number

If it is assumed that the Prandtl number remains constant based on the data shown in Attachment 2, Hilbert's equation shown in Equation 22 can be modified by taking the natural log of both sides. The resulting equation is shown in Equation 26,

$$\ln(Nu) = b * ln(Re) + c \qquad (26)$$

where b = m and $c = \ln(CPr^{1/3})$. Therefore, according to Equation 26, the plot of the natural log of the Nusselt Number versus the natural log of the Reynolds Number should be linear. A visual representation of this data is shown in Figure 3.

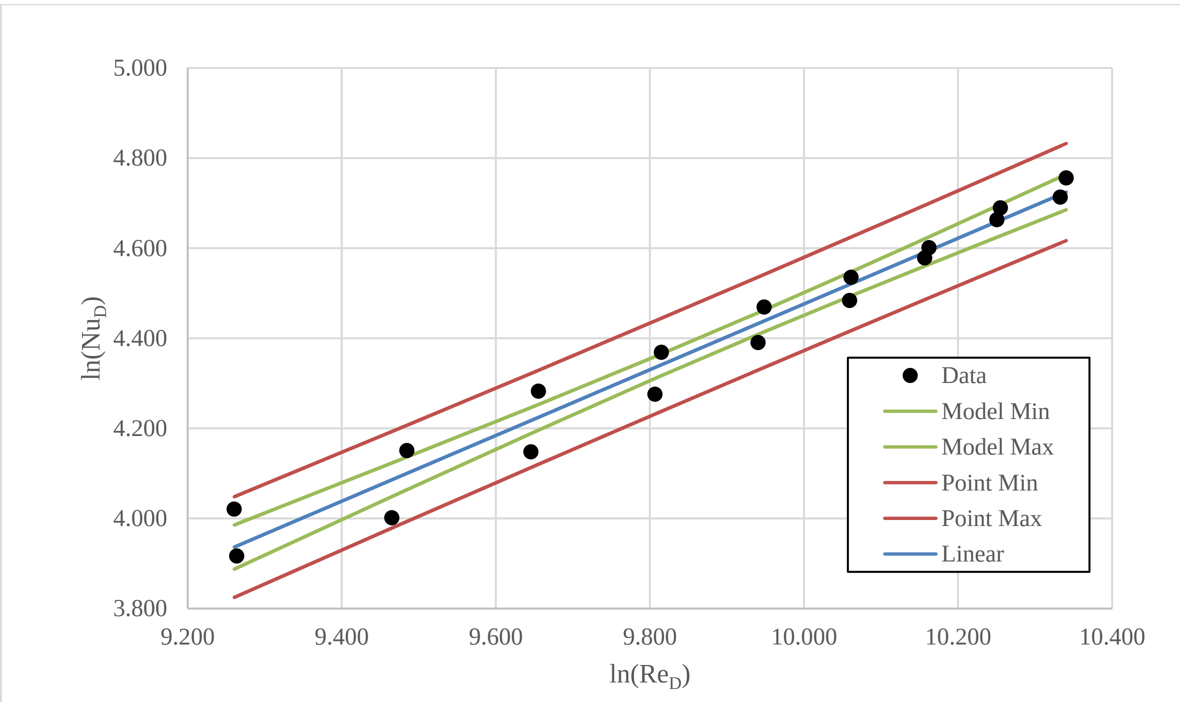

Figure 3. Plot of Natural Log of Nusselt Number Versus Natural Log of Reynolds Number.

The data analysis for this is shown in Attachment 3. As seen from this attachment, the P-value that corresponds to the slope of the linear model has a value of 1.54344E-13. Because the P-value is much less than 0.05 this is an acceptable value for the slope. The P-value for the intercept is also much less than 0.05 with a value of 1.58053E-07, so all together, this linear fit is an accurate fit for the data. The extraordinarily low P-values mean that there was very little variance among the data and therefore a very small tolerance for the model's b and c parameters. The model min and model max differs by no more than 0.1 meaning that the model for the Nusselt number should vary by no more than $e^{0.1} \approx 0.9$. This low tolerance shows that the data was measured very precisely, but not necessarily accurately

CLOSURE

The heat transfer coefficient was calculated with changing velocities (Using Equation 8) in two different ways, by increasing fan speed and by decreasing fan speed (Figure 1). Since only 5 minutes was afforded between each reading to allow the copper rod to reach steady-state temperature, instead of 30 minutes as required by 3 time constants, one of the methods overshot the actual values of h and the other method undershot it. Hence, the actual relationship lies in between these two datasets. The maximum uncertainty in the convection coefficient was 10.8 $\frac{\text{W}}{\text{m}^2\text{C}}$ while the typical uncertainty was 6 $\frac{\text{W}}{\text{m}^2\text{C}}$. The difference between the two datasets is greater than the typical uncertainty, and this can be explained by the bias uncertainty of not allowing enough time for the copper rod to reach steady-state temperatures.

The Nusselt Number value was then calculated based on the convection coefficient previously calculated using Equation 20. Literature values for the Nusselt number were also calculated based on the experimental Reynolds Numbers and Prandtl Numbers. When comparing the experimental Nusselt Numbers to the theoretical Nusselt Numbers, it was determined that the difference was not within the calculated uncertainty due to various assumptions that were made. A linear model was then made of the natural log of the Nusselt Number versus the natural log of the Reynolds Number due to the relationship of the Nusselt Number and the Reynolds Number shown in Equation 22. Because there was an extraordinarily low P-value for both the slope coefficient and the intercept, a linear fit for this data was made appropriately. This shows that the data was acquired with high precision and is a good model of the Hilpert relationship of the Reynolds Number and the Nusselt Number, when the Prandtl Number remains constant.

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Attachment 1. Experiment conditions, calculation of convection coefficient and EPA.

Name:	**Abhay Dalmia**			**Section:**	A7						
Heat Conduction			**Radiation**			**Rod Geometry**			**Ambient Conditions**		
const.	0.246	W/C	emissivity	0.3		Qelec	50	Watts	T_amb	18.5	C
coeff.	0.016		S-B const	5.67E-08	W/m^2-K^4	Hcyl	0.2147	m	P_amb	746	mmHg
						Dcyl	0.0318	m			
						Acyl	0.0214	m^2			
	EXPERIMENTAL DATA AND CALCULATIONS										
	Fan	Vel	Velocity	T_inf	T_avg	T_Sdev	Q-cond	Q-rad	Q-leak	Q-conv	**h**
		(ft/min)	(m/s)	(C)	(C)	(C)	(W)	(W)	(W)	(W)	**W/m^2C**
1	4.6	3010	15.291	14.024	33.515	1.70029	9.5	0.745	10.240	39.760	**95.203**
2	4.1	2780	14.122	14.420	35.065	1.71733	9.7	0.797	10.473	39.527	**89.355**
3	3.6	2540	12.903	14.003	36.296	1.73854	10.0	0.864	10.884	39.116	**81.887**
4	3.1	2305	11.709	14.033	37.663	1.69942	10.2	0.922	11.097	38.903	**76.833**
5	2.6	2070	10.516	14.186	39.232	1.67097	10.3	0.986	11.298	38.702	**72.116**
6	2.1	1820	9.246	13.869	41.160	1.67325	10.7	1.084	11.773	38.227	**65.371**
7	1.6	1565	7.950	14.256	43.632	1.64525	10.9	1.184	12.088	37.912	**60.229**
8	1.1	1330	6.756	13.944	46.650	1.64234	11.5	1.336	12.860	37.140	**52.997**
9	0.6	1075	5.461	14.076	50.404	1.59154	12.1	1.514	13.571	36.429	**46.800**
10	0.6	1090	5.537	14.580	53.521	1.72158	13.0	1.653	14.624	35.376	**42.397**
11	1.1	1320	6.706	14.208	50.505	1.80677	12.8	1.514	14.275	35.725	**45.935**
12	1.6	1565	7.950	14.419	46.768	1.83602	12.0	1.325	13.334	36.666	**52.898**
13	2.1	1820	9.246	14.322	43.501	1.80837	11.4	1.175	12.604	37.396	**59.812**
14	2.6	2065	10.490	14.368	40.921	1.80846	10.9	1.056	11.978	38.022	**66.828**
15	3.1	2310	11.735	14.236	38.782	1.79176	10.6	0.965	11.544	38.456	**73.117**
16	3.6	2535	12.878	14.436	37.094	1.77542	10.2	0.884	11.058	38.942	**80.210**
17	4.1	2775	14.097	14.670	35.729	1.73392	9.9	0.816	10.679	39.321	**87.141**
18	4.6	3000	15.240	14.432	34.562	1.76380	9.8	0.775	10.565	39.435	**91.425**
Averages				14.249	41.389	1.729	10.853	1.088	11.941	38.059	68.920

UNCERTAINTY ANALYSIS						
	Diameter				**Length**	
Kc	2.00957524			Kc	2.009575237	
SSD	6.2973E-05	m		SSD	0.000762648	m
N	50			N	50	
Ua	1.7897E-05	m		Ua	0.000216742	m
Ub	0.00001	m		Ub	0.0044	m
Total U	**2.0501E-05**	**m**		**Total U**	**0.004405335**	**m**
	T infinity				**T Surface**	
Kc	2.10981558			Kc	2.160368656	
SSD	0.225429	C		SSD	1.83602	C
N	18			N	14	
Ua	0.11210321	C		Ua	1.060086387	C
Ub	1.5	C		Ub	1.5	C
Total U	**1.50418321**	**C**		**Total U**	**1.836786092**	**C**

		Uj	(units)	dA/dxj	(Uj*dA/dxj)^2	
Diameter	0.0317676	0.00002050	m	6.745E-01	1.912E-10	
Length	0.2147	0.004	m	9.980E-02	1.933E-07	
Area	0.02	0.0004398736	m^2			
Typical		(units)	Uj	(units)	dh/dxj	(Uj*dh/dxj)^2
T_inf	14.25	C	1.504	C	2.411	13.157
T_s	41.389	C	1.836786	C	-2.411	19.618
Q_e	50.00	W	0.590	W	1.720	1.028
Q_l	11.94	W	0.060	W	-1.720	0.011
A	2.14E-02	m^2	4.40E-04	m^2	-3054.309	1.805
h	**65.445**	**W/m^2 C**	**5.968**	**W/m^2 C**	Combined and Expanded	

Max		(units)	Uj	(units)	dh/dxj	(Uj*dh/dxj)^2
T_inf	14.25	C	1.504	C	4.448	44.766
T_s	33.515	C	1.837	C	-4.448	66.752
Q_e	50.00	W	0.590	W	2.422	2.040
Q_l	14.62	W	0.060	W	-2.422	0.021
A	2.14E-02	m^2	4.399E-04	m^2	-3999.390	3.095
h	**85.696**	**W/m^2 C**	**10.802**	**W/m^2 C**	Combined and Expanded	

Attachment 2. Comparison to literature (theoretical) models.

File: Correlation, created by Z. Zhang, February 11, 2006						Revision September 20, 2006					
Modified by Abhay Dalmia, A7, 6/6/2017											
Ambient Properties:					**Cylinder Geometry:**						
T(ambient) =		18.5	C		Diameter	0.03177	m				
P(ambient) =		99.46	kPa		Length =	0.194	m				
		346.56	P*M/R		HXfer Area	0.01932	m^2				

Computation of Re and Nu from experiments

					dyn	thermal	spec heat			**exp**	**exp**
Velocity	T_inf	T_s	T_film	density	viscosity	conduct	cp	Prandtl	Reynold	**Nusselt**	**h**
m/s	C	C	K	kg/m^3	N s/m^2	W/m-K	J/kg-K	Pr	Re	Nu	W/m^2-C
15.291	14.024	33.515	296.9	1.167	1.83E-05	0.0260	1006	0.708	30979	116.25	95.20
14.122	14.420	35.065	297.9	1.163	1.83E-05	0.0261	1007	0.708	28444	108.79	89.36
12.903	14.003	36.296	298.3	1.162	1.84E-05	0.0261	1007	0.708	25925	99.58	81.89
11.709	14.033	37.663	299.0	1.159	1.84E-05	0.0262	1007	0.708	23428	93.23	76.83
10.516	14.186	39.232	299.9	1.156	1.84E-05	0.0262	1007	0.707	20931	87.29	72.12
9.246	13.869	41.160	300.7	1.153	1.85E-05	0.0263	1007	0.707	18315	78.93	65.37
7.950	14.256	43.632	302.1	1.147	1.86E-05	0.0264	1007	0.707	15615	72.42	60.23
6.756	13.944	46.650	303.4	1.142	1.86E-05	0.0265	1007	0.707	13165	63.47	52.99673
5.461	14.076	50.404	305.4	1.135	1.87E-05	0.0267	1007	0.706	10520	55.73	46.79992
5.537	14.580	53.521	307.2	1.128	1.88E-05	0.0268	1007	0.706	10554	50.22	42.39744
6.706	14.208	50.505	305.5	1.134	1.87E-05	0.0267	1007	0.706	12908	54.68	45.93461
7.950	14.419	46.768	303.7	1.141	1.86E-05	0.0266	1007	0.707	15464	63.29	52.89789
9.246	14.322	43.501	302.1	1.147	1.86E-05	0.0264	1007	0.707	18163	71.92	59.81208
10.490	14.368	40.921	300.8	1.152	1.85E-05	0.0263	1007	0.707	20764	80.66	66.82764
11.735	14.236	38.782	299.7	1.157	1.84E-05	0.0262	1007	0.707	23386	88.55	73.117
12.878	14.436	37.094	298.9	1.159	1.84E-05	0.0262	1007	0.708	25778	97.36	80.20983
14.097	14.670	35.729	298.3	1.162	1.84E-05	0.0261	1007	0.708	28315	105.95	87.14139
15.240	14.432	34.562	297.6	1.164	1.83E-05	0.0261	1007	0.708	30741	111.39	91.42539

Comparison with literature				N =	18				
	exp								
Reynold	Nusselt	Prandtl							
Re	Nu	Pr	Lit_1	Dif^2	Lit_2	Dif^2	Lit_3	Dif^2	
30962	116.18	0.708	98.08	327.78	102.54	186.09	102.44	188.92	
28428	108.73	0.708	92.76	255.03	97.26	131.44	97.34	129.66	
25911	99.52	0.708	87.37	147.59	91.84	58.94	92.14	54.52	
23415	93.18	0.708	81.91	126.97	86.26	47.86	86.81	40.65	
20920	87.24	0.707	76.32	119.24	80.45	46.01	81.28	35.46	
18305	78.89	0.707	70.27	74.32	74.08	23.17	75.25	13.27	
15607	72.38	0.707	63.77	73.99	67.11	27.68	68.69	13.62	
13158	63.43	0.707	57.59	34.10	60.39	9.27	62.36	1.14	
10513.94	55.696	0.7062	50.50	26.97	52.56	9.83	55.02	0.46	
10548.41	50.192	0.7059	50.60	0.17	52.66	6.08	55.11	24.18	
12901.32	54.648	0.7062	56.93	5.19	59.65	24.97	61.66	49.23	
15455.28	63.258	0.7066	63.40	0.02	66.70	11.85	68.29	25.35	
18153.29	71.882	0.7069	69.91	3.89	73.69	3.26	74.88	8.96	
20752.79	80.615	0.7072	75.94	21.88	80.05	0.32	80.90	0.08	
23372.96	88.501	0.7074	81.82	44.61	86.16	5.47	86.71	3.21	
25764.17	97.302	0.7076	87.05	105.01	91.52	33.48	91.82	30.03	
28299.35	105.89	0.7077	92.49	179.68	96.99	79.27	97.07	77.72	
30723.56	111.33	0.7079	97.58	188.97	102.05	86.17	101.96	87.84	
			std dev	9.82	std dev	6.63	std dev	6.60	
			%	11.79	%	7.96	%	7.93	
Lit_1 =	Equation on page 15-3 of Lab Manual, Fand and Keswani (1972).								
Lit_2 =	Hilpert correlation, Eq. (7.55b) on page 410 of Incropera & DeWitt (2002) - cite book.								
Lit_3 =	Churchill and Bernstein, Eq. (7.57) on page 411 of Incropera & DeWitt (2002) - cite book.								

Attachment 3. Linear regression analysis for experimental data.

	File name: Linear Regression Analysis			Created by Z. Zhang			Created 1/25/2006				
							Modified 9/21/2012				
	Demonstration of linear regression analysis and error bound										
	N =	18									
	DF =	16									
	k_c =	2.12	(coverage factor of t-statis)								
	y (ln Nu)	x (ln Re)	Linear	SXX	stand. error	u_model	u_point	Mod-min	Mod_Max	Point_min	Point_max
1	4.756	10.341	4.724699	0.208791	0.00096296	0.03933	0.10782	4.68537	4.76403	4.61688	4.83251
2	4.689	10.255	4.662401	0.138069	0.00072953	0.03482	0.10625	4.62758	4.69722	4.55615	4.76865
3	4.601	10.162	4.594717	0.077751	3.8404E-05	0.03045	0.1049	4.56426	4.62517	4.48981	4.69962
4	4.535	10.061	4.5208	0.031528	0.00020494	0.02663	0.10386	4.49417	4.54743	4.41694	4.62466
5	4.469	9.948	4.438556	0.004208	0.00093818	0.02408	0.10323	4.41448	4.46263	4.33532	4.54179
6	4.369	9.815	4.341105	0.004713	0.00075572	0.02413	0.10324	4.31698	4.36523	4.23786	4.44435
7	4.282	9.655	4.224729	0.052032	0.00332931	0.02839	0.10432	4.19634	4.25312	4.12041	4.32905
8	4.151	9.485	4.100136	0.159057	0.00253787	0.03622	0.10672	4.06392	4.13635	3.99342	4.20685
9	4.020	9.260	3.936439	0.388267	0.00706107	0.04894	0.11168	3.88750	3.98538	3.82476	4.04812
10	3.916	9.264	3.938828	0.384199	0.00050277	0.04875	0.11159	3.89008	3.98757	3.82723	4.05042
11	4.001	9.465	4.085784	0.175128	0.00710952	0.03725	0.10707	4.04853	4.12304	3.97871	4.19286
12	4.148	9.646	4.217608	0.056579	0.00487674	0.02876	0.10442	4.18884	4.24637	4.11318	4.32203
13	4.276	9.807	4.33504	0.005923	0.00353594	0.02425	0.10327	4.31079	4.35929	4.23177	4.43831
14	4.390	9.940	4.432713	0.003234	0.00180394	0.02398	0.10321	4.40873	4.45670	4.32950	4.53592
15	4.484	10.059	4.519491	0.030894	0.00129064	0.02657	0.10384	4.49292	4.54606	4.41565	4.62333
16	4.578	10.157	4.590581	0.074623	0.00014902	0.03021	0.10483	4.56037	4.62079	4.48575	4.69541
17	4.663	10.251	4.659079	0.134708	1.5007E-05	0.03459	0.10618	4.62449	4.69367	4.55290	4.76526
18	4.713	10.333	4.719065	0.201795	3.6144E-05	0.03891	0.10766	4.68016	4.75797	4.61140	4.82673
	4.391	9.884	2.131499	0.047354							
	y_average	x-average	SXX	SEE							

Regression Analysis Output

SUMMARY OUTPUT							
Regression Statistics							
Multiple R	0.98456491						
R Square	0.96936807						
Adjusted R S	0.96745358						
Std. Error	0.04735353						
Observations	18						
ANOVA							
	df	*SS*	*MS*	*F*	*Sig. F*		
Regression	1	1.135374	1.13537	506.3308	2E-13		
Residual	16	0.035878	0.00224				
Total	17	1.171252					
	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
Intercept	-2.82220263	0.320765	-8.7984	1.58E-07	-3.5022	-2.1422	-3.5022	-2.1422
X Variable 1	0.72983883	0.032435	22.5018	1.54E-13	0.6611	0.7986	0.66108	0.7986

Attachment 4. U_B uncertainty for ruler and caliper

Caliper U_B calculation: comparison to gage block

Gage block U_B = 0.2 µm
Gage block rate length = 60 mm
Gage block measured length by caliper = 60.010 mm
Measurement difference = 60-60.011 mm
= 0.01 mm

$$U_{B\_caliper} = \sqrt{U_{B_{gage\,block}}^2 + (measurement\ difference)^2}$$

$$U_{B\_caliper} = \sqrt{(0.2\ \mu m)^2 + (0.01\ mm)^2} = \mathbf{0.01\ mm}$$

Ruler U_B calculation: comparison to caliper

Caliper U_B = 0.01 mm
Arbitrarily chose ruler length = 3 cm
Ruler length measured by caliper = 2.956 cm
Measurement difference = 3-2.956 cm
= 0.044 cm

$$U_{B\_ruler} = \sqrt{U_{B_{caliper}}^2 + (measurement\ difference)^2}$$

$$U_{B\_ruler} = \sqrt{(0.01\ mm)^2 + (0.044\ cm)^2} = \mathbf{4.4\ mm}$$