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ME 4056 Section A7

SUBJECT: Lab 6 – Vapor Pressure

## INTRODUCTION

The vapor pressure experiment was performed at the Georgia Institute of Technology Mechanical Engineering Thermal Laboratory at the George W. Woodruff School of Mechanical Engineering on June 13, 2017. The objective of the lab was to determine the enthalpy of vaporization,  $h_{fg}$ , for refrigerant R-134a at 45°C. The enthalpy was determined by observing the relationship between temperature and pressure of R-134a, determining the specific volume of the R-134a vessel, and applying the data and results to the Clausius-Clapeyron equation. The Clausius-Clapeyron equation requires the refrigerant to be a saturated mixture, so the quality of R-134a was also determined at every data point.

## APPARATUS AND UNCERTAINTY

### Apparatus.

The data collected in this experiment required the use of a sample pressure vessel attached to a bourdon tube pressure gage and submerged in a tank of water heated with a heater, specifically a Whitey Model HDF4-110 110mL nominal volume, Ashcroft Duragage Model Q-8451 with 50 kPa markings, and Lauda E100 Immersion Thermostat respectively. The bourdon pressure gage measured the pressure of the R-134a inside the sample vessel as the vessel was heated in the water tank. The temperature of the water tank and therefore the sample vessel was measured with the platinum resistance thermometer (RTD) in conjunction with the multimeter, specifically the Agilent 33401A Multimeter with the Burns Engineering Model 12001-A-12-6-2-A/LT14. The RTD outputted the temperature through the multimeter as a resistance value, requiring the manufacturer provided conversion equation discussed later in the report. The specific volume of the sample vessel was determined by using the R-134a density, the mass of the empty sample vessel, and the mass of the full sample vessel. The full sample vessel was weighed with a triple beam balance, specifically, Ohaus Heavy Duty Solution Balance Model 1119D 0 to 20 kg with 1 g markings. An Omega K-Type thermocouple was used, not for data collection, but to verify the water in the tank had reached the heater set point temperature. To measure the ambient conditions a thermometer was used to measure the temperature, specifically a VWR General Purpose Glass Thermometer, Cat. #89095-598; - 20 °C to 110 °C w/ 1 °C markings, to measure the pressure a barometer was used, specifically O-N Ins. Aneroid Barometer; 500-780 mmHg w/ 5 mmHg markings. Table 1 lists the equipment used in this experiment and their associated uncertainties. The references for the uncertainty values are displayed below the table.

**Table 1.** Uncertainty of all utilized measurement devices.

Generic ID	Commercial ID	$U_A$	$U_B$	$U_C$
Bourdon Tube Pressure Gage	Ashcroft Duragage Model Q-8451 w/ 50 kPa markings	25 kPa <sup>(1)</sup>	5 kPa <sup>(6)</sup>	25 kPa
Triple Beam Balance	Ohaus Heavy Duty Solution Balance, Model 1119D; 0 to 20 kg w/ 1 g markings Can be zeroed with knob	0.5 g <sup>(1)</sup>	Neg. <sup>(2)</sup>	0.5 g
Platinum Resistance Thermometer and Multimeter	Agilent 33401A Multimeter w/ Burns Engineering Model 12001-A-12-6-2-A/LT14	0.013 °C <sup>(7)</sup>	0.058 °C <sup>(7)</sup>	0.059°C
Multimeter	Agilent 34401A Multimeter	0.005 Ω <sub>(1)</sub>	0.023 Ω <sup>(4)</sup>	0.030 Ω
Thermocouple	Omega K-Type	0.1 °C <sup>(1)</sup>	1.5 °C <sup>(3)</sup>	1.5 °C
Heater	Lauda E100 Immersion Thermostat	N/A	N/A	N/A
Sample Vessel	Whitey Model HDF4-110 110 mL nominal volume	N/A	N/A	N/A
Thermometer	VWR General Purpose Glass Thermometer, Cat. #89095-598; -20 °C to 110 °C w/ 1 °C markings	0.5 °C <sup>(1)</sup>	1 °C <sup>(5)</sup>	1.1 °C
Barometer	O-N Ins. Aneroid Barometer; 500-780 mmHg with 5 mmHg markings	330 Pa <sub>(1)</sub>	N/A	330 Pa

<sup>(1)</sup> By inspection; <sup>(2)</sup> Zeroing; <sup>(3)</sup> Omega Engineering Inc, (2008); <sup>(4)</sup> Agilent, (2001); <sup>(5)</sup> H-B Instrument Company, (2009); <sup>(6)</sup> Jeter, (1996); <sup>(7)</sup> Error Propagation Analysis

### Uncertainty.

Three types of uncertainty are found for each apparatus. Type A uncertainty,  $U_A$ , is the uncertainty associated with error by the user. Type B uncertainty,  $U_B$ , is uncertainty associated with the device. Type C uncertainty,  $U_C$ , is found by relating Type A and Type B uncertainties, using Equation 1.

$$U_C = \sqrt{U_A^2 + U_B^2} \quad (1)$$

### Type A Uncertainty.

The  $U_A$  of the bourdon tube pressure gage, triple beam balance, thermometer, and barometer is 25 kPa, 0.5 g, 0.5 °C, and 330 Pa respectively, determined by taking half of the smallest graduation for each device. The  $U_A$  of the multimeter and thermocouple is 0.005  $\Omega$  and 0.1 °C respectively, determined by observing the maximum fluctuation between the recorded data and the numerical range the devices output at that data point. The heater and sample vessel were established to have no uncertainty in regards to this experiment prior to conducting the experiment by Matthew Perrella and his lecture material. It is important to list these devices in the uncertainty table for apparatus clarification and accurate experimental set-up in the event of a recreation.

### Type B Uncertainty.

The  $U_B$  of the bourdon tube pressure gage and thermometer is 5 kPa and 1°C respectively, determined by the uncertainty data provided by the respective manufacturers: Jeter (1996) and H-B Instrument Company (2009). The multimeter  $U_B$  is 0.023  $\Omega$ , determined by the manufacturer's, Agilent (2001), uncertainty data specifying Equation 2,

$$U_B = 0.01\% \text{ of reading} + 0.001\% \text{ of range} \quad (2)$$

where the maximum data reading was 129.1  $\Omega$  and the range is 1000  $\Omega$ . The  $U_B$  of the thermocouple is 1.5°C, determined by the manufacturer's, Omega Engineering Inc. (2008), uncertainty data specifying the  $U_B$  with a 95% confidence interval is  $\frac{(2.2*2)}{3} = 1.5$ . The  $U_B$  with a 95% confidence interval is used as opposed to the provided  $U_B$  in the 99.6% confidence interval in order to keep the device uncertainty as conservative as possible, within reason. The  $U_B$  of the triple beam balance is negligible because the device was zeroed before being used in the experiment.

### Error Propagation Analysis for $T_{RTD}$ , the temperature measured with the RTD.

The temperature measured with the RTD can be found with Equation 3,

$$T_{RTD} = 9.9725 \times 10^{-4} R^2 + 2.3074 R - 240.77 \quad (3)$$

where  $R$  is the resistance outputted by the multimeter attached to the RTD. Consequently,  $T_{RTD}$  can be expressed as  $T_{RTD}=f(R)$ , so the error propagation analysis is carried out as Equation 4,

$$U_{T_{RTD}} = \sqrt{\left(\frac{\partial T_{RTD}}{\partial R} U_R\right)^2} \quad (4)$$

where  $U_{RTD}$  is the uncertainty for the RTD and  $U_R$  is the uncertainty for the multimeter. Applying Equation 4 to Equation 3, the resulting uncertainty expression is Equation 5,

$$U_{RTD} = 2.5U_R \quad (5)$$

where the influence coefficient (2.5) is determined by the manufacturer's, Burns Engineering (2004), provided uncertainty data. To determine  $U_A$  for the RTD, use the multimeter's  $U_A$  for  $U_R$ . The same is done for  $U_B$  for the RTD with the multimeter's  $U_B$  for  $U_R$ . Detailed calculations shown below.

$$U_{AT_{RTD}} = 2.5 * 0.005 = 0.013 \text{ }^\circ\text{C}$$

$$U_{BT_{RTD}} = 2.5 * 0.023 = 0.058 \text{ }^\circ\text{C}$$

The RTD is a more accurate and reliable thermometer to use, as opposed to the thermocouple, because the combined uncertainty for the RTD is significantly smaller than the combined uncertainty for the thermocouple. The thermocouple served as a good double check for the heater, but would have not served as the best device for measuring experimental temperature data.

## PROCEDURE

The following procedure was used to complete the experiment. Using these steps, several data points were recorded and calculations were made as discussed in the following section.

1. Record the ambient temperature and pressure.
2. Weigh the sample vessel with pressure gage on the triple beam balance.
3. Set the water tank heater to 30°C, wait approximately 10 minutes for the tank to reach the set temperature.
4. Place sample vessel into the water tank, making sure the pressure gage is not submerged.
5. Wait 5-8 minutes for the sample vessel to reach thermal equilibrium with the water tank. Ensure the water tank has reached the heater temperature by measuring the water with the thermocouple.
6. Place the RTD in the water tank and record the multimeter reading.
7. Record the sample vessel pressure with the pressure gage reading.

8. Increase the water tank heater temperature in arbitrary intervals between 30°C and approximately 75°C until 11 measurements are recorded (it isn't important to have uniform temperature intervals as long as a representative range has been accounted for).
9. Repeat steps 3-7 at each temperature point.

### **Justification of Points Chosen**

The data points chosen to take were at 11 varying intervals between 30°C and 74°C. This range was chosen to encompass the target measurements required to calculate the enthalpy of vaporization at 45°C and to capture the saturated mixture at various quality values. Ultimately, the data hopes to compare the accuracy of the theoretical, experimental, and literature comparison methods for calculating the enthalpy of vaporization for a saturated mixture.

## DATA ANALYSIS AND FINDINGS

### Part 1. Calculation of $h_{fg}$ , the Enthalpy of Vaporization

In order to calculate  $h_{fg}$ , the Clausius-Clapyeron equation is used. This is shown below in Equation 6,

$$h_{fg} = v_{fg}T \left( \frac{\partial P}{\partial T} \right)_{sat} \quad (6)$$

where  $h_{fg}$  is the Enthalpy of Vaporization in J/kg,  $v_{fg}$  is the difference in specific volume of the saturated vapor and saturated liquid in  $m^3/kg$ , T is the temperature in Kelvins, and  $\left( \frac{\partial P}{\partial T} \right)_{sat}$  is the partial derivative of pressure with respect to temperature of the saturated liquid.

It can be assumed that  $v_g \gg v_f$  because  $v_f$  is so small in comparison to  $v_g$ . It can also be assumed that  $v_g$  is an ideal gas. Using these assumptions,  $v_{fg}$  can be simplified as shown below in Equation 7,

$$v_{fg} \approx \frac{RT}{P} \quad (7)$$

where  $v_{fg}$  is the difference in specific volume of the saturated vapor and saturated liquid in  $m^3/kg$ ,  $R = 81.5 \frac{J}{kg \cdot K}$  is the specific gas constant for Refrigerant 134a, T is the temperature in Kelvins, and P is the pressure in Pa.

Substituting Equation 7 into Equation 6, Equation 8 is formed, which is shown below,

$$h_{fg} = \frac{RT^2}{P} \left( \frac{dP}{dT} \right)_{sat} \quad (8)$$

where  $h_{fg}$  is the Enthalpy of Vaporization in J/kg,  $R = 81.5 \frac{J}{kg \cdot K}$  is the specific gas constant for Refrigerant 134a, T is the temperature in Kelvins, P is the pressure in Pa, and  $\left( \frac{dP}{dT} \right)_{sat}$  is the derivative of pressure with respect to temperature of the saturated liquid.  $\left( \frac{dP}{dT} \right)_{sat}$  is shown as a total derivative instead of a partial derivate (as shown in Equation 1) because  $P_{sat}$  is only a function of  $T_{sat}$  when they are within the vapor dome.

Rearranging Equation 8 gives Equation 9 shown below,

$$\int \frac{1}{P} dP = \int \frac{h_{fg}}{RT^2} dT \quad (9)$$

when all the values are within the vapor dome.

When integrating Equation 9, it can be assumed that  $h_{fg}$  is constant because it fluctuates very little with small changes in temperature.

Thus, a linear relationship can be made as shown below in Equation 10,

$$\ln(P) = \frac{-h_{fg}}{RT} + C \rightarrow \ln(P) = \frac{C_1}{T} + C_0 \quad (10)$$

where  $P$  is the pressure in Pa,  $h_{fg}$  is the Enthalpy of Vaporization in J/kg,  $R = 81.5 \frac{\text{J}}{\text{kg}\cdot\text{K}}$  is the specific gas constant for Refrigerant 134a,  $T$  is the temperature in Kelvins, and  $C$  is a constant in Pa.

From this linear relationship, we can determine  $\left(\frac{\partial P}{\partial T}\right)_{sat}$  to insert into Equation 6.

Taking the derivative of Equation 10, we get Equation 11,

$$\left(\frac{\partial P}{\partial T}\right)_{sat} = -\frac{C_1 P}{T^2} \quad (11)$$

where  $P$  is the pressure in Pa,  $\left(\frac{\partial P}{\partial T}\right)_{sat}$  is the partial derivative of pressure with respect to temperature of the saturated liquid,  $C_1$  is the linear regression slope from Equation 10.

Inserting this into Equation 1 finally gives us Equation 12,

$$h_{fg} = v_{fg} \left(\frac{-C_1 P}{T}\right) \quad (12)$$

where  $h_{fg}$  is the Enthalpy of Vaporization in J/kg,  $v_{fg}$  is the difference in specific volume of the saturated vapor and saturated liquid in  $\text{m}^3/\text{kg}$ ,  $C_1$  is the linear regression slope from Equation 10,  $P$  is the pressure in Pa and  $T$  is the temperature in Kelvin.

## Part 2. Error in calculation of $h_{fg}$ , the Enthalpy of Vaporization

An EPA was done on Equation 12 above, as shown in Equation 13,

$$U_{h_{fg}} = \sqrt{\left(U_P \frac{\partial h_{fg}}{\partial P}\right)^2 + \left(U_{C_1} \frac{\partial h_{fg}}{\partial C_1}\right)^2 + \left(U_{v_{fg}} \frac{\partial h_{fg}}{\partial v_{fg}}\right)^2 + \left(U_T \frac{\partial h_{fg}}{\partial T}\right)^2} \quad (13)$$

The partial derivatives are as follows:

$$h_{fg} = v_{fg} \left(-\frac{C_1 P}{T}\right) \quad (14)$$

where  $h_{fg}$  is the Enthalpy of Vaporization in J/kg,  $v_{fg}$  is the difference in specific volume of the saturated vapor and saturated liquid in  $\text{m}^3/\text{kg}$  and is a function of temperature,  $C_1 = -2692.8 \text{ Pa}\cdot\text{K}$  is a constant derived in Figure 1,  $P$  is the pressure in Pa, and  $T$  is the Temperature in Kelvins

$$\frac{dh_{fg}}{dv_{fg}} = \left(-\frac{C_1 P}{T}\right) \quad (15)$$

where  $\frac{dh_{fg}}{dv_{fg}}$  is the derivate derivative of Enthalpy of Vaporization with respect to the difference in specific volume of the saturated vapor and saturated liquid in  $\text{Pa}^2$ ,  $C_1 = -2692.8 \text{ Pa}\cdot\text{K}$  is a constant derived in Figure 1,  $P$  is the pressure in Pa, and  $T$  is the Temperature in Kelvins

$$v_{fg} \approx \frac{RT}{P} \quad (16)$$

where  $v_{fg}$  is the difference in specific volume of the saturated vapor and saturated liquid in  $\text{m}^3/\text{kg}$ ,  $R=81.5 \text{ J/kg}\cdot\text{K}$  is the specific gas constant for Refrigerant 134a,  $T$  is the temperature in Kelvins, and  $P$  is the pressure in Pa.

$$\frac{dv_{fg}}{dT} = \frac{R}{P} \quad (17)$$

where  $\frac{dv_{fg}}{dT}$  is the derivate derivative of the difference in specific volume of the saturated vapor and saturated liquid with respect to Temperature in  $\text{m}^3/\text{kg}\cdot\text{K}$ ,  $R=81.5 \text{ J/kg}\cdot\text{K}$  is the specific gas constant for Refrigerant 134a, and  $P$  is the pressure in Pa

$$\frac{dh_{fg}}{dC_1} = -v_{fg} \left(\frac{P}{T}\right) \quad (18)$$

where  $\frac{dh_{fg}}{dC_1}$  is the derivate of Enthalpy of Vaporization with respect to  $C_1 \text{ J/kg}\cdot\text{K}\cdot\text{Pa}$ ,  $v_{fg}$  is the difference in specific volume of the saturated vapor and saturated liquid in  $\text{m}^3/\text{kg}$  and is a



function of temperature, P is the pressure in Pa, and T is the Temperature in Kelvins

$$\frac{dh_{fg}}{dP} = v_{fg} \left( -\frac{C_1}{T} \right) \quad (19)$$

where  $\frac{dh_{fg}}{dP}$  is the derivative of Enthalpy of Vaporization with respect to pressure in J/kg-Pa,  $v_{fg}$  is the difference in specific volume of the saturated vapor and saturated liquid in m<sup>3</sup>/kg and is a function of temperature, and T is the Temperature in Kelvins

$$\frac{dh_{fg}}{dT} = v_{fg} \left( \frac{C_1 P}{T^2} \right) \quad (20)$$

where  $\frac{dh_{fg}}{dT}$  is the derivative of Enthalpy of Vaporization with respect to temperature in J/kg-K,  $v_{fg}$  is the difference in specific volume of the saturated vapor and saturated liquid in m<sup>3</sup>/kg and is a function of temperature, and T is the Temperature in Kelvins

However,  $U_{v_{fg}}$  is considered negligible as it was taken from literature (tables) and  $U_T = 0$  for the case where temperature is given as 45°C, but this uncertainty in T still exists in the experimental data values.

Therefore, Equation 21 below shows the total uncertainty in experimental enthalpy values.

$$U_{h_{fg}} = \sqrt{\left( U_P v_{fg} \left( \frac{C_1}{T} \right) \right)^2 + \left( U_{C_1} v_{fg} \left( \frac{P}{T} \right) \right)^2 + \left( U_T v_{fg} \left( \frac{C_1 P}{T^2} \right) \right)^2} \quad (21)$$

Values were chosen for the above parameters in order to maximize the uncertainty.

Using the values  $v_{fg} = 0.025875 \frac{m^3}{kg}$ ,  $C_1 = -2692.82$  (from part 3 of this report),  $T = 303.0956$  K,  $P = 2320660$  Pa and uncertainties of  $U_P = 25000$  Pa,  $U_{C_1} = SEE \cdot Kc = 28.872 \cdot \text{TINV}(0.05,9) = 65.314$  (as seen in Attachment 2) and  $U_T = 0.059K$ , Equation 21 is computed to solve for the uncertainty of the enthalpy of vaporization as

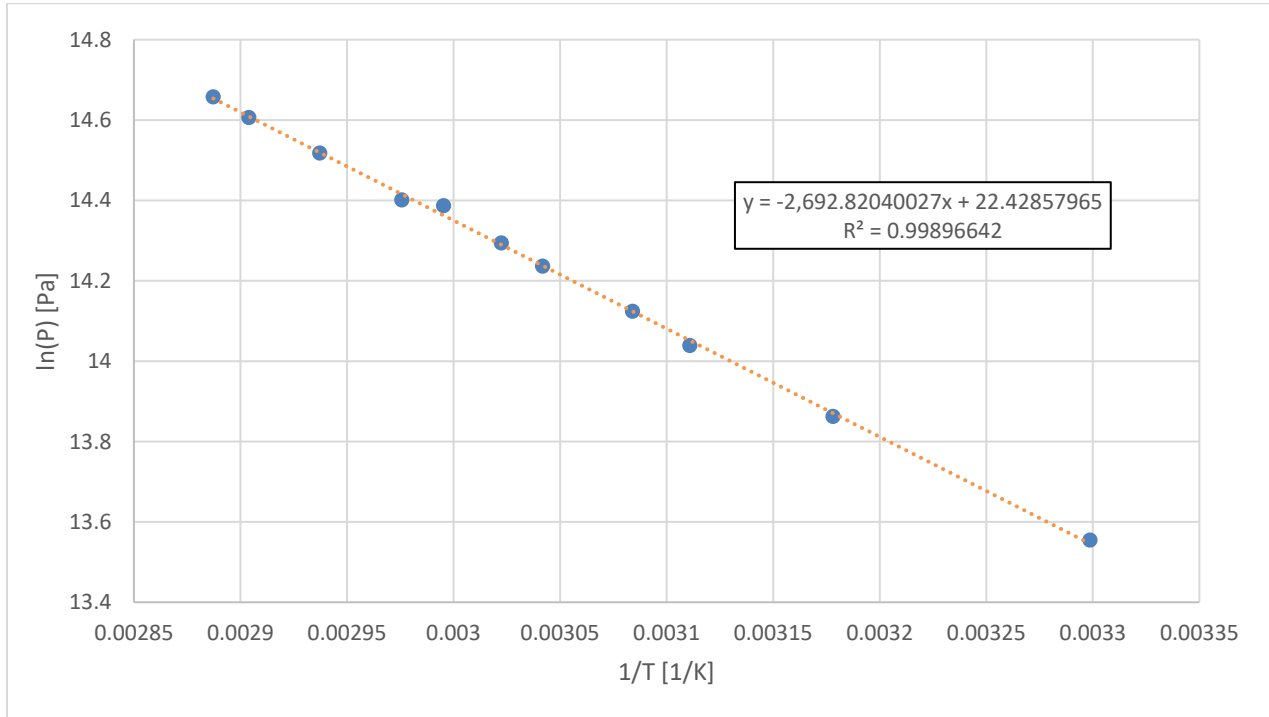
$$U_{h_{fg}} = 14.16 \text{ kJ/kg}$$

Equation 22 is the total uncertainty for the case that temperature is given as 45C,

$$U_{h_{fg}} = \sqrt{\left(U_P v_{fg} \left(\frac{C_1}{T}\right)\right)^2 + \left(U_{C_1} v_{fg} \left(\frac{P}{T}\right)\right)^2} \quad (22)$$

The uncertainty of this equation is also  $U_{h_{fg}} = 14.16$  kJ/kg as it turns out that the temperature uncertainty is negligible due to the high accuracy of the RTD used.

### Part 3. Experimental determination of $C_1$ and enthalpy



**Figure 1.** Plot of the natural log of Pressure against the reciprocal of Temperature

Attachment 2 shows the regression analysis of the plot shown in Figure 1. Because of this plot's incredibly low P-value of 9.5E-15 and extraordinarily large  $R^2$  value of 0.999, it can be concluded that this data fits a linear regression. This proves that Equation 5 is valid and can be used for solving  $h_{fg}$ . Additionally, a quadratic regression with terms up to  $1/T^2$  yields a p-value of greater than 0.5 (Attachment 3). Hence, a linear regression is the best fit.

Using Equation 10 and the slope of the graph shown in Figure 1, it can be concluded that the  $C_1$  constant in Equation 10 is equal to -2692.8 Pa·K. Since  $C_1 = \frac{-h_{fg}}{R}$ , this equation can be

substituted back into Equation 7, in order to solve for  $h_{fg}$  at each corresponding temperature and pressure. This is shown in Equation 6 below,

$$\left(\frac{dP}{dT}\right)_{sat} = \frac{h_{fg}P}{RT^2} = -\frac{C_1P}{T^2} \quad (23)$$

This then gets substituted back into Equation 6, giving the final relationship shown below in Equation 24,

$$h_{fg} = v_{fg} \left(-\frac{C_1P}{T}\right) \quad (24)$$

where  $h_{fg}$  is the Enthalpy of Vaporization in J/kg,  $v_{fg}$  is the difference in specific volume of the saturated vapor and saturated liquid in  $\text{m}^3/\text{kg}$  and is a function of temperature,  $C_1 = -2692.8 \text{ Pa}\cdot\text{K}$  is a constant derived in Figure 1,  $P$  is the pressure in Pa, and  $T$  is the Temperature in Kelvins.

Attachment 1 shows the experimental and theoretical  $h_{fg}$  values calculated at each corresponding temperature and pressure using Equation 12. When comparing the experimental  $h_{fg}$  calculated using Equation 12 against the literature values shown in the DuPont table, the RMS error was calculated to be 4.09 kJ/kg. Since this error is well within the uncertainty of enthalpy of vaporization calculated as 14.16 kJ/kg, the results are correct/accurate within the limits of uncertainty of this experiment. Because this value is within the calculated uncertainty of  $h_{fg}$ , it can be concluded that the Clausius-Clapyeron equation is valid for calculating Enthalpy of Vaporization.

In order to successfully calculate these values, many assumptions were made in the process, some of which may drastically affect the data. These assumptions include, neglecting the saturated liquid specific volume, assuming the gas acts as an ideal gas, and assuming the enthalpy of vaporization stays the same with changes in temperature. Some of these assumptions are a stretch and explain the high variability within the data.

Attachment 1 shows the quality at each point. This is calculated as shown below in Equation 25,

$$x = \frac{v - v_f}{v_{fg}} \quad (25)$$

where  $v = 1.375 \frac{\text{mL}}{\text{g}}$  is the specific volume of the refrigerant,  $v_f$  is the specific volume of the saturated liquid in  $\frac{\text{mL}}{\text{g}}$ , and  $v_{fg}$  is the difference in specific volume of the saturated vapor and

saturated liquid in  $\frac{\text{mL}}{\text{g}}$ .

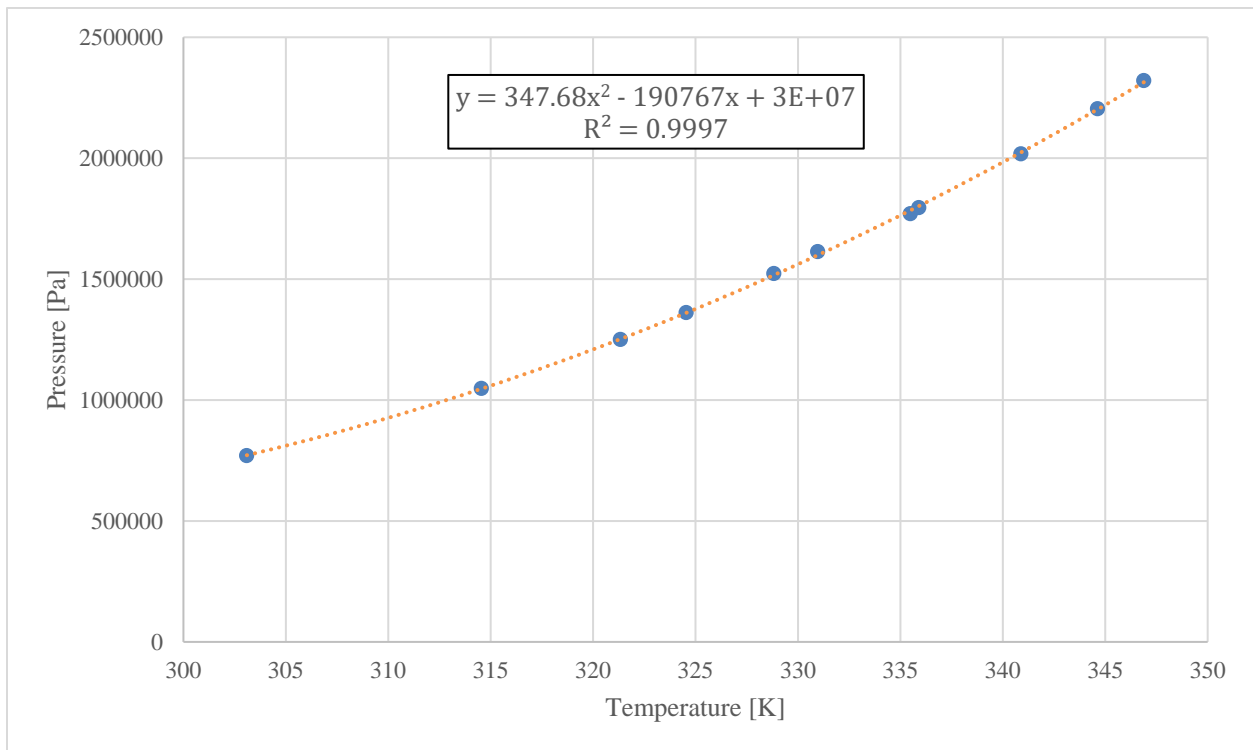
Attachment 1 shows that the quality at each point ranges from about 0.03 to 0.05. Because these values are between 0 and 1, this shows that the refrigerant's specific volume always lies between the saturated liquid specific volume and the saturated vapor specific volume. This means that the refrigerant is always a two-phase mixture. This assumption is then valid and held true for the experiment and is not a source of error.

#### Part 4. Second Method for calculating $h_{fg}$ , the Enthalpy of Vaporization

The pressure at each data point was not directly measured. What was actually measured was the gage pressure within the Pressure Gage. Converting this to absolute pressure is shown below in Equation 26,

$$P_{abs} = 1.01 \cdot P_{gage} + P_{amb} \quad (26)$$

where  $P_{abs}$  is the absolute pressure of the refrigerant in Pa,  $P_{gage}$  is the gage pressure of the refrigerant inside the Pressure Gage in Pa, and  $P_{amb} = 98660$  Pa is the ambient pressure of the lab room.



**Figure 2.** Plot of the Pressure against Temperature

Attachment 4 shows the regression analysis done on the data shown in Figure 2. Because the P-Values for a quadratic fit are well below 0.05 (1.1E-8 and 4.2E-8) and the R<sup>2</sup> value is extraordinarily close to 1 (0.9997), it can be concluded that the data fits a quadratic curve. Equation 27 below shows the relation of pressure to temperature within the vapor dome,

$$P = 347.6789742 \cdot T^2 - 190766.5833 \cdot T + 26652226.1 \quad (27)$$

where P is pressure in Pa and T is temperature in Kelvins.

Taking the derivative of Equation 27 gives Equation 28 shown below,

$$\frac{dP}{dT} = 695.36 \cdot T - 190766.5833 \quad (28)$$

where  $\frac{dP}{dT}$  is the derivative of pressure with respect to temperature in Pa/K, P is pressure in Pa, and T is temperature in Kelvins.

Attachment 5 shows Equation 28 along with Equation 12 being used to solve for the experimental Enthalpy of Vaporizations. Because this method of solving for  $h_{fg}$  requires less assumptions being made, this experimental data should be much closer to the theoretical data. The RMS error between the theoretical data the experimental data using this method was calculated to be a 6.2 kJ/kg error. This is higher error than the other method; however, there was one data point that stood out as having a very large error. After removing this outlier, the RMS difference significantly drops to a 3.8 kJ/kg error. This low error proves that the Clausius-Clapyeron equation is valid for calculating Enthalpy of Vaporization.

This method (which doesn't make some of the assumptions made by the other method), however, is very close to the first method that assumes among other things, that  $v_{fg} = v_g$  and the ideal gas law. It can be concluded then that the assumptions were valid and held true.

## Part 5. $h_{fg}$ , the Enthalpy of Vaporization at $T = 45^\circ\text{C}$

**Table 2: Calculation of  $h_{fg}$  at 45C**

vf @ 45	vg @ 45	vfg @ 45	P @ 45	$h_{fg}$ @ 45	theoretical $h_{fg}$
0.8881	17.39	16.5019	1160	162.02	157.7

The table above shows the calculations for solving  $h_{fg}$  at  $45^\circ\text{C}$ . There were two methods for solving for this value, the method shown in Part 3 and the method shown in Part 4. The first method gave an experimental value of 162.02 kJ/kg, and the second method gave an experimental value of 159.92. The literature value for the Enthalpy of Vaporization at  $45^\circ\text{C}$  is equal to 157.7 kJ/kg. The first method has an error of 4.3 kJ/kg and the second method has an error of 2.2 kJ/kg.

The variable that affected the uncertainty most was the regression constant  $C_1$ . It contributed to about 65% of the uncertainty, while the pressure contributed to the remaining 35%. The uncertainty due to temperature was very low and considered negligible. The uncertainty of the regression constant was high in part due to the high Kc value. Hence, in order to improve the experiment and reduce the uncertainty, more data values/data points have to be collected to increase the DOFs of the data and reduce the Kc value. Additionally, more accurate and sensitive pressure equipment can be used to further reduce the uncertainty.

## CLOSURE

The Enthalpy of Vaporization was calculated with two separate methods. The first one required linearizing the Clausius-Clapyeron equation in Equation 6. In order to do so, many assumptions needed to be made including assuming the gas acted as an ideal gas and assuming that  $h_{fg}$  remained constant with small changes in temperature. This reduced down to the linear relationship shown in Equation 10. This linear relationship was valid because it produced a very low P-value and high  $R^2$  value. Fitting a line through this data gave a slope value of  $-2692.8 \text{ Pa}\cdot\text{K}$  that was then substituted back into Equation 6 and produced the experimental data shown in Attachment 1. This data had an RMS error of 4.09, which is within the uncertainty value of 14.16 and thus shows that this is a valid method for solving the Clausius-Clapyeron equation.

The second method for solving for the Enthalpy of Vaporization required no assumptions to be made other than that pressure was only a function of temperature. After plotting pressure versus temperature, a quadratic relationship was achieved, which gave a low P-value and high  $R^2$  value. The derivative was then taken for this quadratic relationship and was substituted into Equation 6. Since  $v_{fg}$  is just a function of temperature,  $h_{fg}$  could be solved directly by only using the temperature and the pressure-temperature relationship. The corresponding experimental  $h_{fg}$

values are shown in Attachment 5, and these values produced an RMS error of 6.2. This was also less than the uncertainty and proved to be a valid method for solving the Clausius-Clapyeron equation.

Finally, the experimental value for  $h_{fg}$  at 45 degrees Celsius was determined using the two methods described. The first method gave a value of 162.02 kJ/kg, the second method gave a value of 159.92 kJ/kg, and the literature value at 45 degrees Celsius was 157.7 kJ/kg. These values had an error 4.3 and 2.2 kJ/kg respectively, which was far less than the uncertainty of  $h_{fg}$ .

It is also worth noting that the quality of the liquid-vapor at each data point was between 0 and 1. This shows that the refrigerant was always a two-phase liquid-vapor mixture, and the equations that were used were valid.

## REFERENCES

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**Attachment 1. Ambient conditions and data calculation**

<b>Tamb (deg C)</b>	22.5
<b>Pamb (mmHg)</b>	740
Vtank (mL)	110
mass_empty (g)	1764
mass_full (g)	1844
R (kJ/kg-K)	0.0815

Tables showing all intermediate calculation

R [Ohm]	Ref_Tank_T	Water_Tank_T [C]	P_gage (kPa)	P_corr	P_abs [kPa]	P Theory[kPa]	v [mL/g]
111.912	29.9456	30	665	671.65	770.31	769.85766	1.375
122.04	55.6779	55.6	1410	1424.1	1522.76	1516.7602	1.375
116.43	41.39925	41.5	940	949.4	1048.06	1057.0185	1.375
124.65	62.3423	60.7	1655	1671.55	1770.21	1777.9714	1.375
119.1	48.18714	48.3	1140	1151.4	1250.06	1260.7137	1.375
126.76	67.73993	67.3	1900	1919	2017.66	2012.393	1.375
120.36	51.39536	51.1	1250	1262.5	1361.16	1366.2793	1.375
128.22	71.47999	71.2	2085	2105.85	2204.51	2187.8153	1.375
122.88	57.82128	57.7	1500	1515	1613.66	1598.2087	1.375
129.1	73.73632	73.2	2200	2222	2320.66	2298.8268	1.375
124.81	62.75129	62.9	1680	1696.8	1795.46	1794.9035	1.375

Ref_Tank_T	P_abs [kPa]	P Theory[kPa]	vf [mL/g]	vg [mL/g]	vfg [mL/g]	x (quality)	hf_table [kJ/kg]	hg_table [kJ/kg]	hfg_table [kJ/kg]	hfg_exp [kJ/kg]
29.9456	770.31	769.857657	0.84236	26.7167	25.874318	0.02059	90.722	264.1717	173.45	177.077
55.6779	1522.76	1516.76019	0.92888	12.9482	12.019276	0.03712	130.1	274.8898	144.792	149.882
41.39925	1048.06	1057.01849	0.87636	19.2696	18.393231	0.02711	107.83	269.4716	161.645	165.03
62.3423	1770.21	1777.97139	0.95956	10.772	9.8124287	0.04234	140.94	276.6622	135.721	139.42
48.18714	1250.06	1260.71368	0.89989	15.9494	15.049519	0.03157	118.14	272.2111	154.075	157.652
67.73993	2017.66	2012.39302	0.98825	9.25666	8.2684041	0.04677	150.01	277.7384	127.726	131.784
51.39536	1361.16	1366.27931	0.91199	14.5858	13.673825	0.03386	123.23	273.4023	150.17	154.43
71.47999	2204.51	2187.81528	1.0108	8.31368	7.3028859	0.04987	151.32	278.1888	126.864	125.794
57.82128	1613.66	1598.20875	0.93831	12.2151	11.276808	0.03872	133.57	275.49	141.919	148.052
73.73632	2320.66	2298.82676	1.02573	7.7857	6.7599676	0.05167	160.4	278.3242	117.923	121.78
62.75129	1795.46	1794.90348	0.9616	10.6501	9.6885136	0.04267	141.61	276.7603	135.148	139.453

Table only showing final calculated columns used to analyze the data:

Ref_Tank _T	P_abs [kPa]	P Theory[kPa]	x (quality)	hfg_table [kJ/kg]	hfg_exp [kJ/kg]
29.9456	770.31	769.857657	0.02059	173.45	177.07702
55.6779	1522.76	1516.76019	0.03712	144.7916	149.88166
41.39925	1048.06	1057.01849	0.02711	161.6448	165.03
62.3423	1770.21	1777.97139	0.04234	135.7208	139.42034
48.18714	1250.06	1260.71368	0.03157	154.0754	157.65217
67.73993	2017.66	2012.39302	0.04677	127.7257	131.78406
51.39536	1361.16	1366.27931	0.03386	150.1698	154.42983
71.47999	2204.51	2187.81528	0.04987	126.864	125.79429
57.82128	1613.66	1598.20875	0.03872	141.9195	148.05234
73.73632	2320.66	2298.82676	0.05167	117.923	121.78011
62.75129	1795.46	1794.90348	0.04267	135.1482	139.45324

**Attachment 2: Linear Regression Output of ln(P) vs 1/T regression**

SUMMARY OUTPUT								
<i>Regression Statistics</i>								
Multiple R	0.99948308							
R Square	0.99896642							
Adjusted R Square	0.99885158							
Standard Error	0.01124809							
Observations	11							
<i>ANOVA</i>								
	<i>df</i>	<i>SS</i>	<i>MS</i>	<i>F</i>	<i>Significance F</i>			
Regression	1	1.10054231	1.10054231	8698.59888	9.4954E-15			
Residual	9	0.00113868	0.00012652					
Total	10	1.10168099						
	<i>Coefficients</i>	<i>Standard Error</i>	<i>t Stat</i>	<i>P-value</i>	<i>Lower 95%</i>	<i>Upper 95%</i>	<i>Lower 95.0%</i>	<i>Upper 95.0%</i>
Intercept	22.4285797	0.08782651	255.373683	1.1016E-18	22.2299023	22.627257	22.2299023	22.627257
X Variable 1	-2692.8204	28.8723901	-93.266279	9.4954E-15	-2758.1343	-2627.5065	-2758.1343	-2627.5065

**Attachment 3. Quadratic Regression Output of ln(P) vs 1/T regression**

SUMMARY OUTPUT								
<i>Regression Statistics</i>								
Multiple R	0.999507							
R Square	0.999015							
Adjusted R Square	0.998769							
Standard Error	0.011646							
Observations	11							
<i>ANOVA</i>								
	<i>df</i>	<i>SS</i>	<i>MS</i>	<i>F</i>	<i>Significance F</i>			
Regression	2	1.100596	0.550298	4057.464	9.41E-13			
Residual	8	0.001085	0.000136					
Total	10	1.101681						
	<i>Coefficients</i>	<i>Standard Error</i>	<i>t Stat</i>	<i>P-value</i>	<i>Lower 95%</i>	<i>Upper 95%</i>	<i>Lower 95.0%</i>	<i>Upper 95.0%</i>
Intercept	23.76721	2.12998	11.15842	3.72E-06	18.85547	28.67895	18.85547	28.67895
X Variable 1	-3562.76	1383.282	-2.57559	0.032841	-6752.62	-372.908	-6752.62	-372.908
X Variable 2	141105.8	224317.8	0.629044	0.546858	-376172	658383.6	-376172	658383.6

**Attachment 4. Quadratic regression output of P vs. T regression**

SUMMARY OUTPUT								
<i>Regression Statistics</i>								
Multiple R	0.99987206							
R Square	0.99974413							
Adjusted R Square	0.99968016							
Standard Error	8585.3805							
Observations	11							
<i>ANOVA</i>								
	<i>df</i>	<i>SS</i>	<i>MS</i>	<i>F</i>	<i>Significance F</i>			
Regression	2	2.304E+12	1.152E+12	15628.9646	4.2862E-15			
Residual	8	589670067	73708758.4					
Total	10	2.3046E+12						
	<i>Coefficients</i>	<i>Standard Error</i>	<i>t Stat</i>	<i>P-value</i>	<i>Lower 95%</i>	<i>Upper 95%</i>	<i>Lower 95.0%</i>	<i>Upper 95.0%</i>
Intercept	26652226.1	1561394.56	17.0695011	1.4096E-07	23051643.8	30252808.4	23051643.8	30252808.4
X Variable 1	-190766.58	9587.02325	-19.898417	4.2402E-08	-212874.3	-168658.87	-212874.3	-168658.87
X Variable 2	347.678974	14.6993151	23.6527329	1.0863E-08	313.782293	381.575656	313.782293	381.575656

**Attachment 5.** Second method of calculating enthalpy

Temp [k]	P [Pa]	vfg [m <sup>3</sup> /kg]	dp/dt	hfg_exp [kJ/kg]	hfg_table [kJ/kg]	(exp- theory) <sup>2</sup>
303.095603	770310	0.02587432	19993.3532	156.7957109	173.450045	277.3668593
328.8279	1522760	0.01201928	37886.5105	149.7378373	144.791614	24.46512213
314.549248	1048060	0.01839323	27957.7365	161.7516367	161.644782	0.011417922
335.492304	1770210	0.00981243	42520.657	139.9777613	135.720774	18.12193801
321.337142	1250060	0.01504952	32677.7524	158.0286165	154.07543	15.62768465
340.889934	2017660	0.0082684	46273.9421	130.4284613	127.725702	7.304905343
324.545356	1361160	0.01367383	34908.6094	154.9166055	150.169759	22.5325516
344.629985	2204510	0.00730289	48874.6163	123.0073151	126.863974	14.8738155
330.971283	1613660	0.01127681	39376.9289	146.9664918	141.919481	25.47231741
346.886316	2320660	0.00675997	50443.574	118.287167	117.922989	0.132625869
335.901292	1795460	0.00968851	42805.0499	139.3040795	135.148191	17.27140601
<b>RMS Error</b>						<b>6.202496885</b>