Pipe Mechanics for Dunetown

Dunetown Department of Water Services

The only engineering requirement of the pipe that has to be satisfied is that the minimum flow rate has to be at least 10,000 gallons of water per minute to satisfy the present demand for water.

The flow rate of the pipe is governed by:

$$Q_{\text{Flow Rate}} = A_{\text{Cross Sectional Area}} \cdot V_{\text{Velocity}}$$

$$[1]\ Q_{\text{Flow Rate}} = \pi \cdot \left(\frac{D_{\text{Diameter}}}{2}\right)^2 \cdot V_{\text{Velocity}}$$

The velocity in the above equation is further governed by a combination of the energy balance equation and head loss equation:

$$z_{\text{Lake Elevation}} = \frac{V^2}{2 \cdot g} \cdot \left(1 + f_{\text{Friction Factor}} \cdot \frac{L_{\text{Length of Pipe}}}{D}\right)$$

$$[2]\ 30 \cdot \text{m} = \frac{V^2}{2 \cdot 9.807 \cdot \text{m} \cdot \text{s}^{-2}} \cdot \left(1 + f \cdot \frac{2000 \cdot \text{m}}{D}\right)$$

Finally, to friction factor in the above equation is modeled by

$$[3] \begin{cases} \dfrac{1}{\sqrt{f}} = -2.0 \log_{10}\left(\dfrac{\varepsilon}{D \cdot 3.7} + \dfrac{2.51 \cdot \gamma}{V \cdot D \cdot \sqrt{f}}\right), & Re = \dfrac{V \cdot D}{\gamma} > 2100 \\ f = \dfrac{64 \cdot \gamma}{V \cdot D}, & Re = \dfrac{V \cdot D}{\gamma} \leq 2100 \end{cases}$$

$$\gamma = \text{Fluid Viscosity} = 1.12 \cdot 10^{-6}\ \text{m}^2 \cdot \text{s}^{-1}$$

$$\varepsilon = \text{roughness} = 2 \cdot 10^{-4}\ \text{m}$$

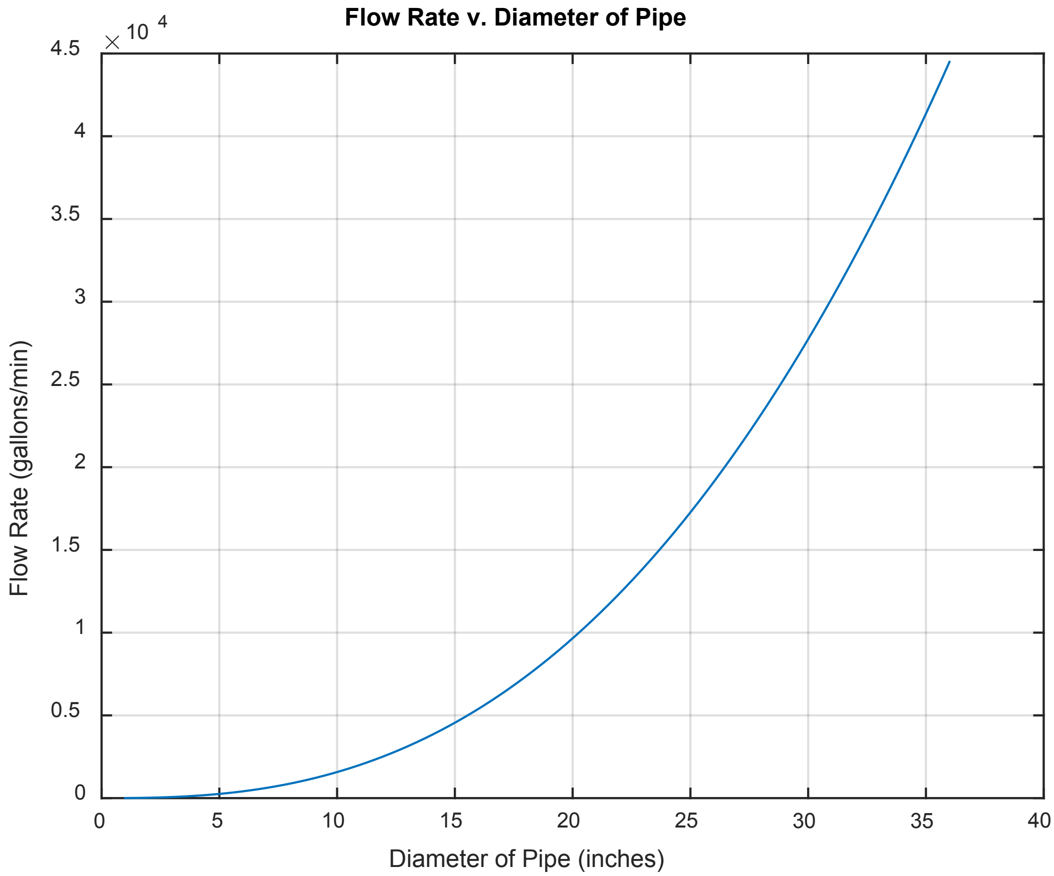
To solve the first equation, two unknown variables velocity (V) and diameter (D) have to be selected which are constrained by equations 2 and 3 above. This is where computational modeling was utilized.

For the modeling, a certain value of diameter was chosen (Example: $D = 0.1$ m). For this value of diameter, the computational **fzero** method built into MATLAB was used to iterate through values of velocity that satisfied equation 3. The **fzero** function would first evaluate the friction factor f using Equation 3 by the **Newton-Raphson** Method. This would then consequently be inserted altogether into Equation 2.

The **fzero** function would continue iterating through different values of velocity, until it satisfies equation 2 with an accepted minimum tolerance of 10 times machine precision. For $D = 0.1$ m, the qualifying velocity found is $V = 1.080244 \text{ m} \cdot \text{s}^{-1}$. Sample iteration summary is shown in Appendix A.

Finally the value of velocity found for that specific diameter solves the 1st equation for the flow rate. This is revaluated for 1000 linearly spaced points between a diameter of 1" and 36". The code calculated the flow rate achieved by each diameter using the method above.

Finally, all the values of diameter and flow rate are plotted on the following graph:



From this plot, it can be extrapolated that to achieve a flow rate of at least 10,000 gallons of water per minute, the pipe must be at least ≈ 20.25" in diameter.

According to **Nominal Pipe Size (NPS)**, a measure by the American Standards Association, the minimum nominal pipe size manufactured is **24"**, about 3.75" larger than needed. This pipe size will allow a flow rate of 15,500 gallons/minute. **Therefore, the pipe recommended by this report is a 24" Steel Pipe.**

Appendix A – Iteration Summary from the **fzero** function

Search for an interval around 4 containing a sign change:

Func-count	a	f(a)	b	f(b)	Procedure
1	4	361.25	4	361.25	initial
3	3.88686	339.652	4.11314	383.461	search
5	3.84	330.885	4.16	392.841	search
7	3.77373	318.666	4.22627	406.285	search
9	3.68	301.745	4.32	425.657	search
11	3.54745	278.533	4.45255	453.77	search
13	3.36	247.141	4.64	494.965	search
15	3.0949	205.619	4.9051	556.094	search
17	2.72	152.64	5.28	648.286	search
19	2.18981	89.2019	5.81019	790.15	search
21	1.44	22.4627	6.56	1013.74	search
22	0.379613	-25.9278	6.56	1013.74	search

Search for a zero in the interval [0.379613, 6.56]:

Func-count	x	f(x)	Procedure
22	0.379613	-25.9278	initial
23	0.533742	-22.2492	interpolation
24	0.533742	-22.2492	interpolation
25	0.983411	-4.98683	interpolation
26	1.09007	0.531221	interpolation
27	1.0798	0.023721	interpolation
28	1.08024	-0.000103	interpolation
29	1.08024	1.7669e-10	interpolation
30	1.08024	7.10543e-15	interpolation
31	1.08024	7.15043e-15	interpolation

Zero found in the interval [0.379613, 6.56]

ans = 1.080244314405411

Appendix B – Number of iterations performed by **fzero**

The **fzero** function in MATLAB uses a combination of several primary root finding methods to implement its own version of finding roots to an equation. One method to know the number of iterations performed by the function is by printing the iteration summary.

The function also outputs a structure containing this information. The 4th output from the function gives the following structure:

```
    intervaliterations: 11
            iterations: 9
             funcCount: 31
             algorithm: 'bisection, interpolation'
               message: 'Zero found in the interval [0.379613, 6.56]'
```

Hence, the number of actual iterations of loops can be calculated by adding the first two numbers in the structure:

```
iterations = struct.intervaliterations + struct.iterations
```

The number of times that the input function is evaluated can be extracted by:

```
funcCount = struct.funcCount
```

Appendix C – Evaluation of Code

ffactor.m

The accuracy of this function was evaluated by checking a few values by hand and then comparing the code's output to the ones calculated algebraically. The Newton-Raphson implementation was also validated by plotting the error in each subsequent evaluation of the root. When plotted on a logarithm curve, the error showed quadratic convergence (straight line with slope of -2) as is expected of the Newton-Raphson method.

pipeflow.m

The accuracy of this function was evaluated by using the test case provided with this task. This accuracy was further evaluated with literature values found on the internet for set diameter sizes of pipes. The function showed slight inaccuracy for some of these values because the functions do not perfectly model the flow in a pipe with these equations. Additional equations will have to be considered in order to refine this model.

diameter.m

The plot was as expected. It was increasing, at an increasing rate due to the rate of flow exponentially increasing with diameter as Equation 1 above suggests.