Extended Essay in Physics

Investigation of the change in viscosity of liquid due to change in temperature by considering the terminal velocity of a falling sphere

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Abstract:

The effect of temperature on viscosity has widespread affects in especially related to engine oils in the automotive industry. **This investigation strives to answer the question "How does the change in temperature of a liquid affect its viscosity by considering the terminal velocity of a falling sphere?"** The theoretical Arrhenius model is evaluated experimentally.

Terminal velocity of a falling sphere is measured at different temperatures from $25°C - 95°C$, for Glycerin, Detergent and Castor Oil. The falling sphere is recorded on video using a camera and on analysis, the distance covered per unit time, when the sphere reaches terminal velocity, is calculated. This is measured by using Photoshop to calculate distance and the camera's frame rate to measure time. Hence, the relationship between terminal velocity and absolute temperature is explored.

A relationship is determined between terminal velocity and viscosity using the theory of major forces: Gravity, Buoyancy, Turbulence and Viscosity (Stokes' Law). This relationship was used to calculate viscosity from terminal velocity. Finally, the relationship between viscosity and absolute temperature is explored.

This investigation has shown evidence that the terminal velocity of the sphere falling through the liquid increases cubically with the absolute temperature. Additionally, the natural log of terminal velocity increases linearly with inverse of temperature.

Viscosity decreases cubically with temperature. Additionally, the natural log of viscosity varies linearly with the inverse of temperature. Finally, this investigation shows that viscosity decreases with temperature as terminal velocity increases with temperature.

This study is merely an attempt to understand the relationship between viscosity and temperature in liquids, and has been limited to three liquids. A continuation of this investigation could lead to generalizations for all liquids and quantifying the effect of turbulent forces on viscosity.

Word Count: 282

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1. Introduction

Viscosity of fluids is an integral part of our lives, having an impact in various products from suntan lotions to lubricant oils. In this essay, I will be investigating the viscosity of liquids and terminal velocity of objects falling/sinking through the liquid. I will be exploring how the viscosity of a liquid (by measuring the terminal velocity of an object falling through that liquid) is affected by temperature.

Lubricants used in engines are very important, in that their purpose is to provide a region of high lubricating index between moving parts to reduce friction. However, engines operate at very high temperatures, ranging anywhere from a 100°C – 1000°C. Thus, formulating the effect of temperature on the viscosity of the lubricant is imperative. According to this effect, lubricants have to be chosen carefully to operate at required temperatures.

Viscosity is informally defined as the "thickness" of a fluid. Generally, the thicker the liquid is, the higher its viscosity. Rather it's defined as the property of a liquid to resist flow, or more appropriately provide resistance to tension.

According to elementary physics, when resistance to motion increases, with other factors remaining same, the speed of the object reduced. Hence, the higher the viscosity of a liquid, the slower an object will sink to the bottom. Therefore, the effect of temperature on viscosity can be measured by its coincidental effect on the velocity of a falling object in that liquid.

2. Theory

There are many forces acting on a sphere as it moves through a liquid. The resolution and analysis of these forces will provide us with viscosity. This viscosity is essentially what I will analyze to fashion a conclusion. For this experiment, I will only examine the forces parallel to the velocity-displacement plane.

2.1 Force of Gravity (Weight)

The force of gravity acts on any mass, and acts towards the center of the Earth. Hence, it acts vertically down, towards the direction of motion of the sphere. Even though the force of gravity changes as distance between the Earth and the sphere changes, it is negligible for a small change and will be considered constant (9.81NKg^{-1}).

Weight of Sphere = $mass \cdot a_{gravity}$

Mass of Sphere $= volume\ of\ a\ sphere \cdot density$

$$= \frac{4}{3}\pi r^3 \cdot \rho_{ball}$$

Where r is radius of the sphere and ρ is the density of the sphere.

Hence,

Weight of Sphere = $\boldsymbol{\frac{4}{3}\pi r^3 \cdot \rho_{ball} \cdot g}$ **(Equation 1)**

2.2 Force of Buoyancy

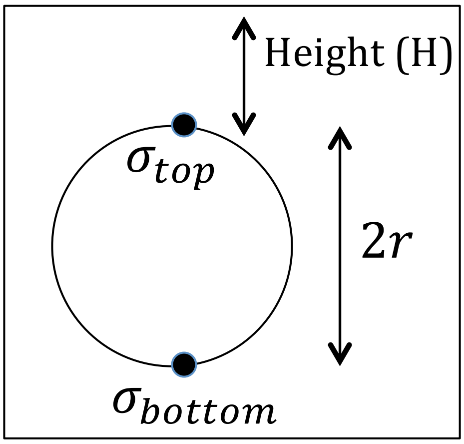


Figure 1 Shows an object moving through a fluid with the differences in pressure created

The force of buoyancy is a force opposing the direction of motion, created due to a difference in pressure between the top of the object and the bottom of the object moving through a liquid. (*See Figure 1*)

$$\text{Pressure in Liquids} = \sigma_{fluid} \cdot h \cdot g$$
$$\sigma_{top} = \sigma_{fluid} \cdot H \cdot g$$
$$\sigma_{bottom} = \sigma_{fluid} \cdot (H + 2r) \cdot g$$

This difference in pressure creates a force vertically upwards that accelerates the sphere upwards. It opposes the weight (force of gravity) of the sphere.

This Buoyant Force is also equal to the weight of the liquid displaced due to the object being fully submerged.

$$F_{buoy} = weight\ of\ liquid\ displaced$$
$$= m_{liquid}\ displaced \cdot g$$ **(Equation 2)**

$$m_{water}\ displaced = v_{water}\ displaced \cdot \sigma_{liquid}$$
Since $v_{liquid}\ displaced = v_{ball}$
$$m_{liquid}\ displaced = v_{ball} \cdot \sigma_{liquid}$$
(Where v is volume)
$$m_{liquid}\ displaced = \frac{4}{3}\pi r^3 \cdot \sigma_{liquid}$$ **(Equation 3)**

Inserting Equation 3 into 2

$$F_{buoy} = m_{liquid}\ displaced \cdot g$$
$$\boldsymbol{F_{buoy} = \frac{4}{3}\pi r^3 \cdot \sigma_{liquid} \cdot g}$$
(Equation 4)

2.3 Force of Turbulence

Turbulence occurs when the magnitude of the velocity of an object exceeds an arbitrary value. The flow does not remain laminar and eddies are formed. At this point, the fluid starts behaving erratically and may change both the magnitude and direction of the velocity vector.

Hence, the force of turbulence may affect the calculation of velocity and hence increase uncertainty. Additionally, since this force is "random" and cannot be easily calculated, the only choice is to make sure that the flow remains laminar.

$$Re = \frac{2r \cdot velocity \cdot \sigma_{liquid}}{\eta}$$
(Equation 5)

Where:
Re = Reynold's Number
r = Radius of Sphere
η = Viscocity

The flow is considered laminar when $Re \ll 1$.

2.4 Force of Viscocity

Viscosity in a liquid is caused due to friction between neighboring fluid particles that move at different velocities. Some force needs to be applied, in order to overcome this friction and keep fluid particles in motion.

Using Stokes' law, the F_{stress} is provided by the weight of a sphere moving through the liquid. As the sphere moves through the fluid, the layers in contact with the solid are dragged along. This constructs a difference in velocity between fluid layers, and hence a frictional force is created that opposes the direction of motion. The magnitude of this frictional force, termed viscous force, is given by Stokes' Law following –

$$F_{visc.} = k \cdot r\eta v$$

Where $k =$ Surface Area Factor – [6π for spheres]
$\eta =$ Viscosity
$v =$ Velocity of object
$r =$ Radius of Sphere

Hence, for a sphere –

$$\boldsymbol{F_{visc.} = 6\pi r \cdot \eta v}$$

(Equation 6)

Interaction of these forces

All these forces described above act on the object moving through the liquid (a sphere in this case). A force diagram allows us to resolve and equate these forces. (*See Figure 2*)

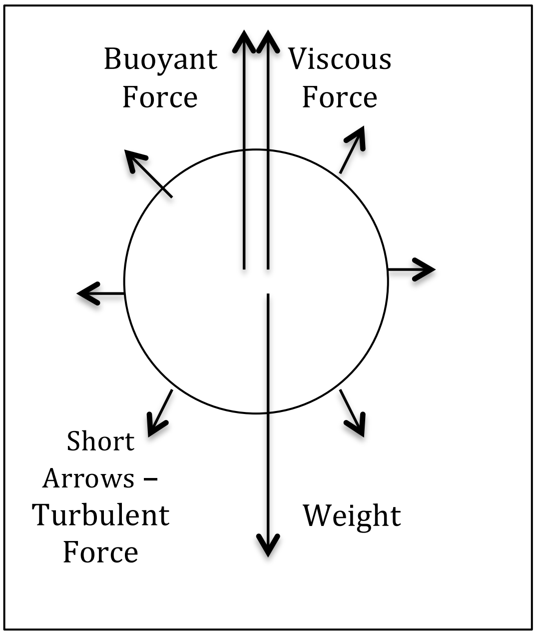


Figure 2 This figure shows all the forces acting on the object as it falls/moves through the liquid

When the sphere starts its fall through the liquid, the downward forces are greater than the upward forces. Hence the sphere accelerates downward. However, as its velocity increases, so does the viscous force. At one point, the upward and downward force equal each other and the object falls with constant velocity. At this condition, we can equate the upward and downward forces. (*See Figure 3*).

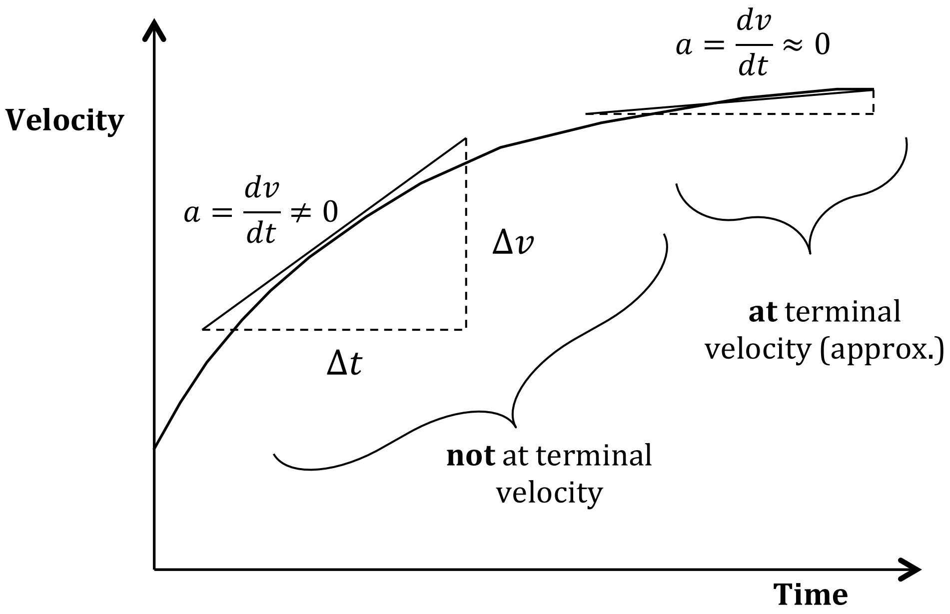


Figure 3 The velocity of the object as time progresses and it falls lower into the fluid

At terminal velocity, acceleration is zero. Since -

$$a = \frac{dv}{dt}$$

Hence, the change in velocity with respect to time must also be zero at terminal velocity.

When the object is at terminal velocity, we can equate the upward and downward forces. Hence –

$$F_{Visc.} + F_{buoy} = F_{weight}$$

Substituting expression of forces from previous part (Equation 1 and 3)

$$F_{Visc.} = \frac{4}{3}\pi r^3 \cdot \rho_{ball} \cdot g - \frac{4}{3}\pi r^3 \cdot \sigma_{liquid} \cdot g$$

$$\boldsymbol{F_{Visc.} = \frac{4}{3}\pi r^3 \cdot g \cdot (\rho_{ball} - \sigma_{liquid})}$$

Finally we have two expressions for the force due to viscosity (one directly above and one from Stokes' Law). Equating the two of them and solving for viscosity will result in the final equation.

$$6\pi r \cdot \eta v = \frac{4}{3}\pi r^3 \cdot g \cdot (\rho_{ball} - \sigma_{liquid})$$

$$\eta = \frac{\frac{4}{3}\pi r^3 \cdot g \cdot (\rho_{ball} - \sigma_{liquid})}{6\pi r v}$$

$$\boldsymbol{\eta = \frac{2r^2 \cdot g \cdot (\rho_{ball} - \sigma_{liquid})}{9v}}$$

(Equation 7)

3. Theoretical Mathematical Hypothesis

The effect of temperature on terminal velocity will be investigated first, since terminal velocity of a falling sphere can be investigated. Since viscosity is further affected by terminal velocity through the relation above (**Equation 7**), the effect of temperature on viscosity can be evaluated, by using terminal velocity.

In liquids, molecules must possess a minimum amount of energy to move past other molecules. The probability that a molecule possess this energy E_a is proportional to $e^{-E_a/RT}$ (Arrhenius Model). This quantifies the probability of mobility. The viscosity of a liquid is inversely proportional to mobility.

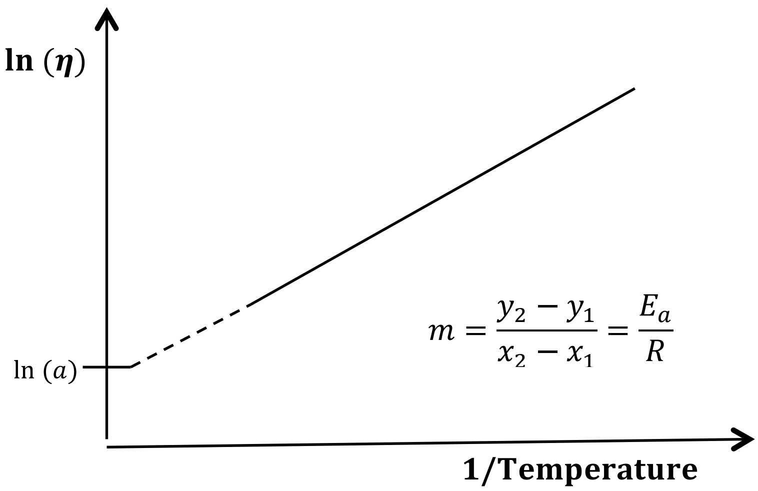
Hence, following that

$$\eta = a \cdot e^{\frac{E_a}{RT}}$$

Where 'a' and 'E_a' are constants **(Equation 8)**

$$\ln(\eta) = \ln(a) + \frac{E_a}{RT}$$

Assuming $y = \ln(\eta)$ and $x = \frac{1}{T}$



This is a relationship between viscosity and temperature. In other words, it shows that as temperature increases, viscosity decrease. In order to arrive to this relationship, the concept of terminal velocity and Stokes' Theorem will be used. Hence, it is important to find a relation between terminal velocity and temperature.

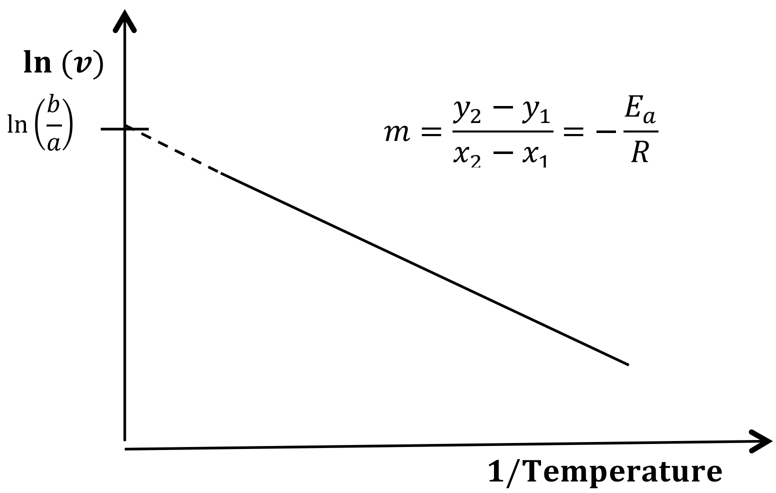
Formulating relationship between temperature and terminal velocity, equate *equation 7 and 8* –

$$a \cdot e^{\frac{E_A}{RT}} = \frac{2r^2 \cdot g \cdot (\rho_{ball} - \sigma_{liquid})}{9} \cdot \frac{1}{v}$$

Assuming $\frac{2r^2 \cdot g \cdot (\rho_{ball} - \sigma_{liquid})}{9} = \text{constant} = b$ and taking natural log both sides –

$$\ln(a) + \frac{E_a}{RT} = \ln(b) - \ln(v)$$

$$\ln(v) = \ln\left(\frac{b}{a}\right) - \frac{E_a}{RT}$$



This relationship shows that as temperature increases, so does terminal velocity. Hence, the terminal velocity of an object moving through a liquid is inversely proportional to the viscosity of the liquid.

4. Apparatus

1. Camera with preferably high frame rate recording for more accurate data. Camera used: Sony Cyber-shot DSC-WX7 (25 frames per second)
2. Micrometer ($\pm$0.005 mm) and Electronic Balance ($\pm$0.001 g) to measure dimensions and mass of object (sphere).
3. Electronic Balance and 10cm^3 graduated cylinder for calculating density of liquids.
4. Heating Element to control temperature – Immersion heater and water bath was used in this experiment. For higher temperatures, water bath was first heated using a gas burner.
5. Tub to heat fluids in
6. Long and graduated cylinder. This is used as the experiment container. (Experiment used 1.2m long cylinder)
7. Insulation/Cotton to prevent heat loss during experiment
8. Thermometer to measure temperature.
9. Retort Stand to hold thermometer in position, during experiment

Diagram of Setup

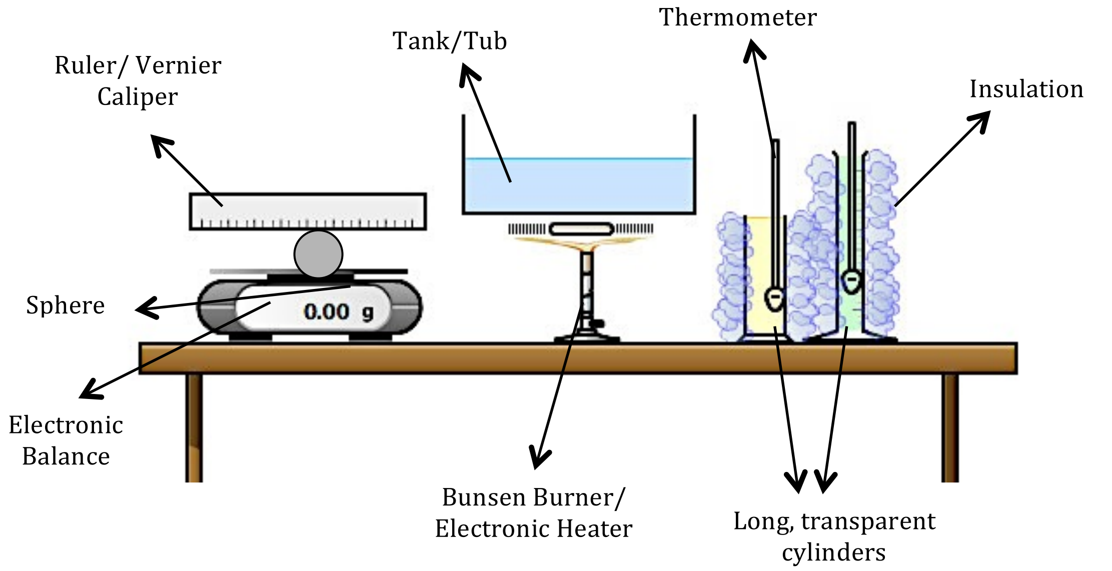


Figure 4 The apparatus used and the rough setup of the three different stages of this experiment; the measuring of dimensions, the heating of liquid and the actual experiment. This diagram was made using the freeware: Chemix 2.0

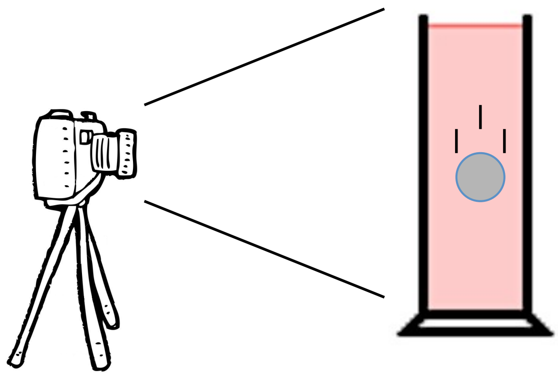


Figure 5 Positioning of the camera. Camera records video as object falls.

5. Manipulation of Variables

Independent Variable:

1. **Temperature of Liquid** – This will be changed by immersing the liquids in a water bath and using a heat source to heat the liquids until the maximum point. Temperature will be recorded using a thermometer. Then, it will be poured into the cylinder and the experiment will be conducted every 5°C from 25°C to 95°C.

Dependent Variable:

1. **Terminal Velocity/Viscosity** – This will be calculated by the *Equation 7* above. The experiment below is used to determine the linear terminal velocity of a sphere falling in a liquid, as the equation demands. This terminal velocity will be introduced into the equation to calculate viscosity. Finally, the relationship between viscosity and temperature will be analyzed.

Controlled Variable:

1. **Turbulent Forces** (*see Force of Turbulence, page 3*) – these forces will be controlled by calculating the Reynolds's number (*Equation 5*) and discounting any values and trials that exceed the laminar-turbulent force boundary.

2. **Radius and Mass of Sphere –** Since the radius and the mass of the sphere are present in the equation, it does not need to be kept constant. However, larger and heavier spheres will not reach terminal velocity, and hence will create unreliability in accuracy. The larger spheres will also increase turbulent flow that leads to unreliability in precision.

3. **Insulation** - The amount of insulation will be kept constant. Insulation will consist of tissue and a thick roll of cotton. Insulation does not have to be necessarily be kept constant, but should be sufficient to maintain slow fall in temperature, so that experiment can be conducted without dire changes.

4. **Room Temperature –**Room temperature also does not have to be constant but has to be controlled in order to prevent rapid fall in temperature.

6. Experimental Methods

There are essentially three main tasks in this experiment. Before the experiment is started, the dimensions of the sphere is measured and recorded. The densities of the liquids are also measured and calculated. Next, the liquid is heated appropriately outside the experiment container. Once the liquid reached desired temperature, it was poured into the experiment container. Then, the spheres were dropped while the camera recorded. Finally, video recording was analyzed for data.

6.1 Varying Temperature of Liquid

The temperature was varied between 25°C and 95°C and the experiment was conducted. To vary the temperature, the liquid was heated in a water bath. This ensured linear heating of the liquid and prevented variations in terms of 'hot spots' and 'cold spots'. The uncertainty in the temperature calculation is ± 0.5°C.

6.2 Recording the Sphere Moving Through the Liquid

The liquid was then transferred from the container to the cylinder. The cylinder was covered with cotton which acts as insulation. Since several trials were carried out at the same temperature, the insulation ensured that variances in temperatures during trials would not cause systematic or random errors.

The mass of the sphere was calculated through an electronic balance (uncertainty of $\pm 0.001\ g$) and the radius, with a micrometer (uncertainty of $\pm 0.01\ mm$)

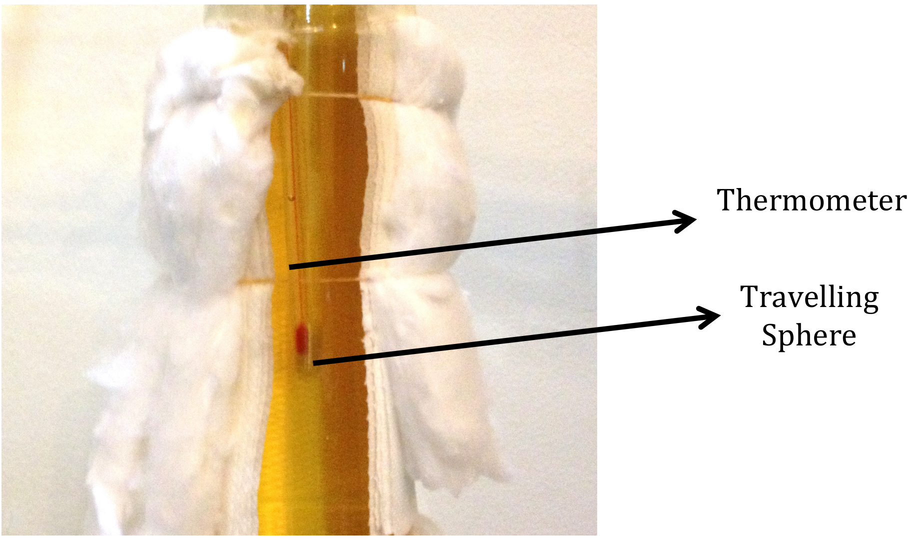


Figure 6 **Falling sphere through castor oil**

6.3 Analyzing video recording for data

The main aim is to find the speed (velocity) of the object moving through the liquid. The camera used takes video at 25 frames per second. This was used in order to keep count of the time. The distance was calculated with the help of pixel to centimeter conversion.

A grid with boxes of 1cm by 1cm was kept adjacent to the cylinder of liquid, at the same distance from the camera. The average pixels equated to the 1cm (found through Adobe Photoshop) will be used to convert the pixels (uncertainty of x°C) covered by the sphere every 5 frames to centimeters.

The velocity calculated $\left[V = \frac{Dist.}{Time}\right]$ for the last 10 frames will be plotted, to infer when and at what speed the sphere reaches terminal velocity. This terminal velocity will further be processed to reflect viscosity by the use of *Equation 7* above.

Figure 7 Measurements fixed next to experiment container

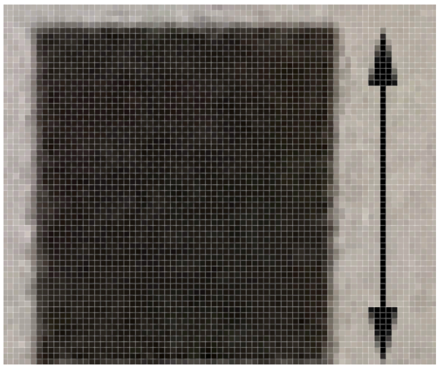


Figure 8 Zoomed and pictured in Photoshop and Illustrator

x Pixels = 1 centimeter

The camera was first calibrated, and pixels were converted to centimeters, to later

7. Data Collection, Processing and Analysis

The idea in this experiment is to vary the temperature and to determine the linear, terminal velocity of the sphere at that temperature. Using this terminal velocity, and the mass, volume and density of the sphere, the viscosity is calculated (*Equation 8*). The change of viscosity is investigated with the change in temperature. A relation is formed by investigating 3 liquids.

7.1 Dimensions of spheres, uncertainties of apparatus

Table 1 Information regarding sphere needed for Eqn. 7

Mass ± 0.001 g		**Diameter** ± 0.005 mm		**Radius**		**Volume (mm^3)** $\left[\frac{4}{3}\pi r^3\right]$		**Density (Kg/m^3)** $\left[\frac{Mass}{Volume}\right]$	
value	%	value	%	value	%	value	%	value	%
0.450	0.222	5.22	0.096	2.61	0.096	74.7	0.287	6025	0.510

Table 2 Trials of Radius Measurement Taken

SPHERE	
5.22	
5.21	
5.23	
5.23	
AVG.	**5.22**

Alcohol Thermometer: $(-10°C$ to $110°C) \pm 0.5°C$
Camera: Sony Cyber-shot DSC-WX7 (25 frames per second)
Therefore, 1 frame=0.04s

Calibration of camera and measurements

Since the camera's view captures the entire length of the cylinder, the further vertically upwards or downwards that is moved from the center focus of the lens, parallax error occurs.

This parallax error can lead to great error if the experiment container is very long. Hence, camera was a calibrated.

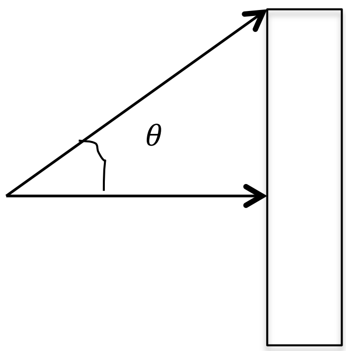


Figure 8 Pictorial representation of parallax error

Figure 9 This figure shows the row of 1cm cube squares that were fixed next to the experiment container. This was removed afterward, but the position of the container and distance from camera were kept constant.

Table 3 Pixels to Centimeters Conversion Chart

Box #	Units
1	4.7
2	4.7
3	4.8
4	4.7
5	4.7
6	4.8
7	4.8
8	4.9
9	4.9
10	4.9
AVG.	**4.8**

Box #	Units
11	4.9
12	5.0
13	5.1
14	5.1
15	5.2
16	5.2
17	5.2
18	5.1
19	5.0
20	4.9
AVG.	**5.1**

Box #	Units
21	5.0
22	4.8
23	4.8
24	4.7
25	4.8
26	4.7
27	4.7
28	4.7
29	4.7
30	4.6
AVG.	**4.8**

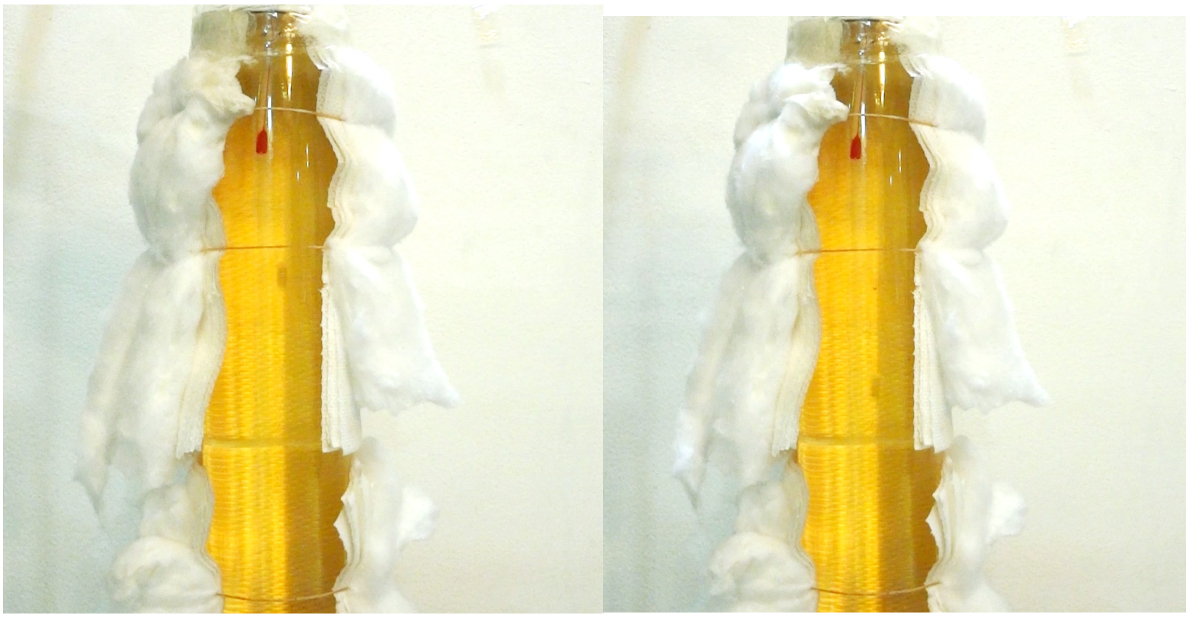
Uncertainty - $\pm 0.1\ Pixels$

Hence, to calculate the distance moved by the sphere per unit time, the experiment container will be divided into three essential vertical parts. For calculation of distance moved in each part, the corresponding pixels to centimeter conversion will be used.

7.2 Velocity of spheres at different temperatures

Step 1: Collect Last 11 frames when Terminal Velocity is Reached

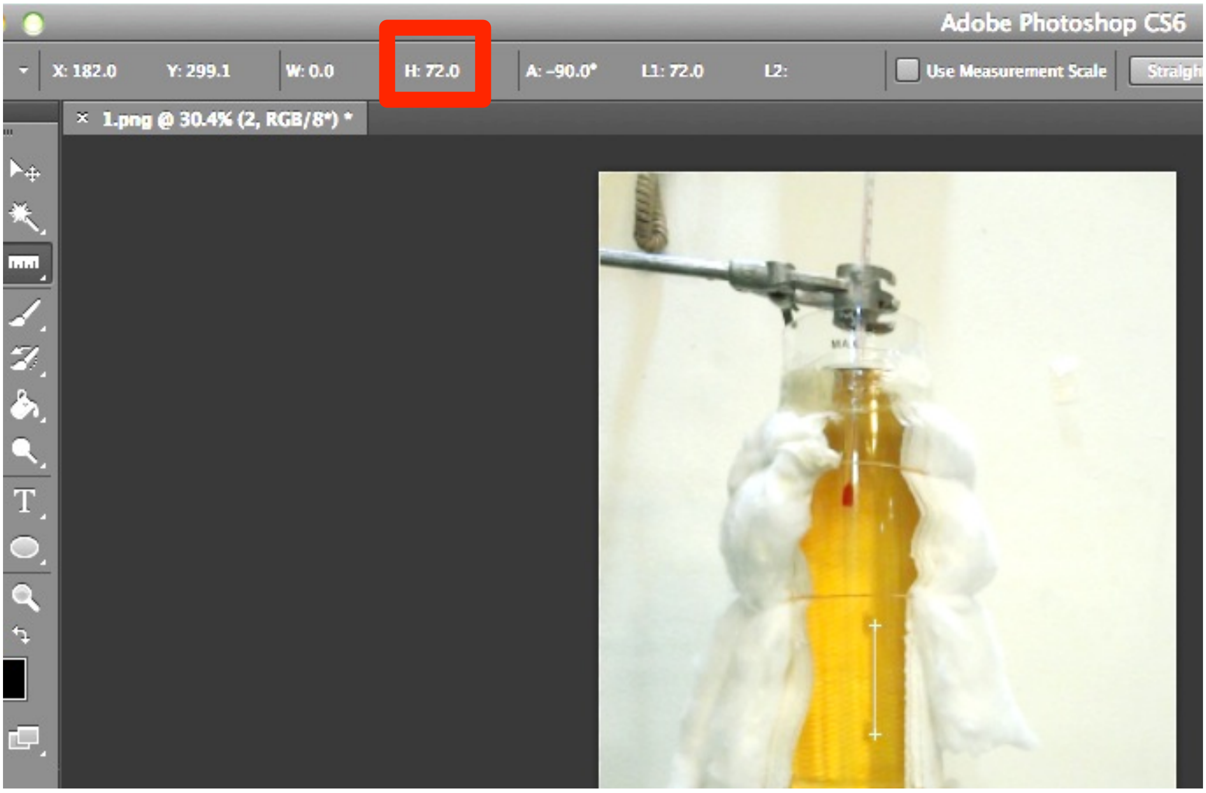
From the many videos collected, I opened each on 'QuickTime Player'. This allows me to view the video frame by frame. Last 11 frames before sphere reached the end was exported as pictures. Examples of the frames that result are –



These two pictures show the video 3 frames apart. 11 frames for each trial and temperature, for each liquid were similarly collected.

Step 2: Merge pictures on Photoshop and Illustrator

Then the frames are merged into one picture, so that difference in the movement of the sphere is shown in one picture. This is done by importing both pictures to Photoshop, and one picture is made the background, whereas the other picture is made the foreground. Then, by using the eraser tool, it erases specific parts of the foreground so the background can be seen through that. We can see both spheres from the two frames in one picture as shown below.



Step 3: Measuring distance between spheres

Then after both spheres are on one picture, the distance between them is calculated using the ruler tool inbuilt into photoshop. This will output the distance in terms of pixels, on the top left corner of the window (outline in red in the picture above).

The figure on the left represents the last 11 frames merged. The distance between each was measured in pixels using the method above.

Step 4: Converting Distance to Velocity

Finally this distance calculated is converted into velocity and the data is produced below. First, pixels have to be converted to centimeters using

Distance (cm)$=\frac{\text{Distance(pixel)}}{4.8}$ (See calibration above).

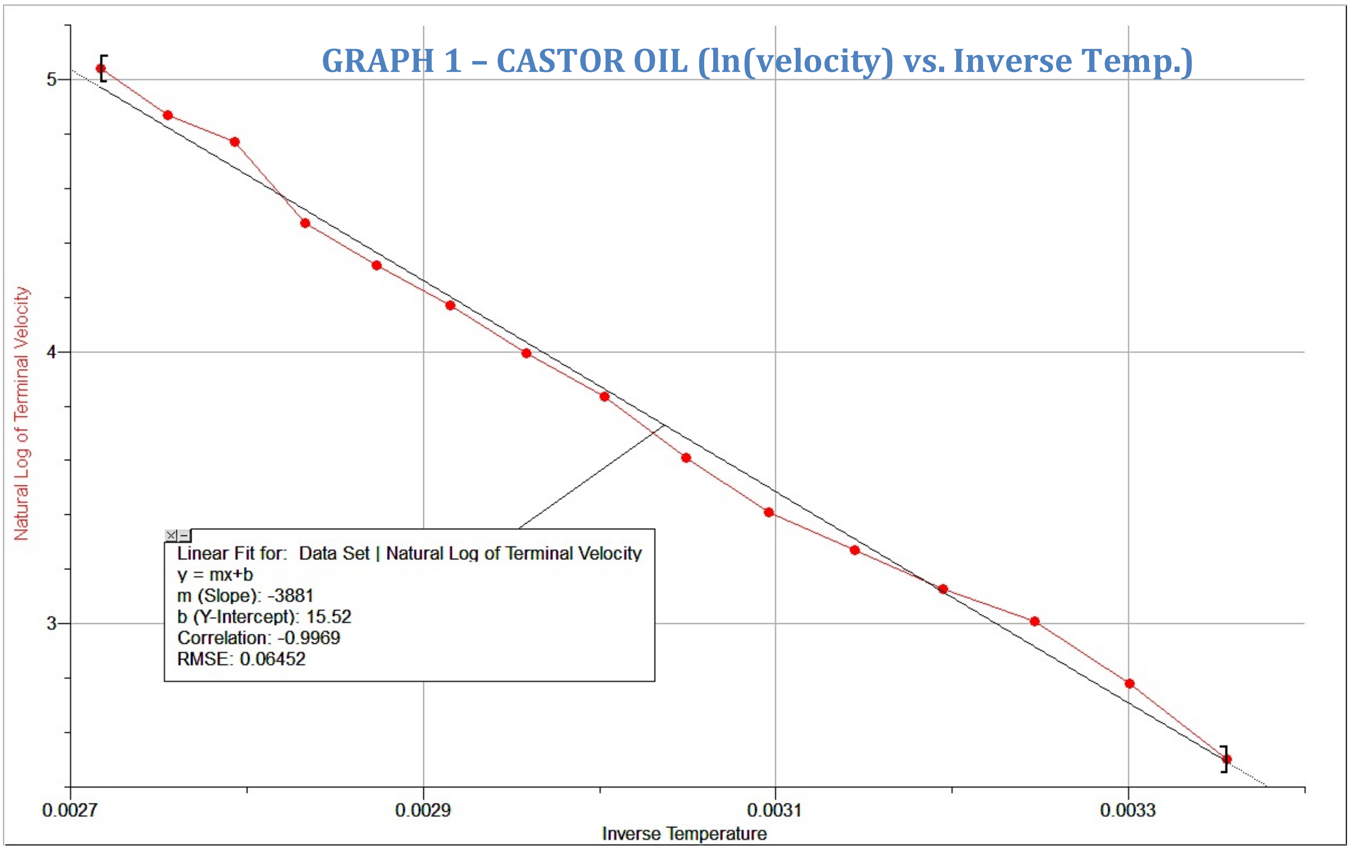
After distance is calculated, this has to be divided 0.04s. Since we are merging consecutive frames, the time between each frame is 0.04s (as mentioned above). Hence Velocity$=\frac{\text{Distance}}{\text{Time}}=\frac{\text{Distance (cm)}}{0.04}$. So, the pure value of pixel achieved from the software is first divided by either 4.8 or 5.1, and then divided by 0.04. Since these are just divisions by constant values, including tables before values were divided by these constants would become excessive and repetitive. Hence, the velocities are directly mentioned below.

CASTOR OIL

Table 4 – Velocity of last 10 frames and Terminal Velocity for Castor Oil

Temp.	Last 10 frames before sphere reaches bottom $(cm \cdot s^{-1})$										AVG.
	1	2	3	4	5	6	7	8	9	10	
25°C	12.2	12.3	12.1	12.2	12.3	12.2	12.2	12.2	12.2	12.2	12.2
	12.3	12.2	12.2	12.2	12.2	12.2	12.1	12.2	12.1	12.2	
	12.2	12.1	12.2	12.1	12.2	12.6	12.1	12.1	12.2	12.1	
30°C	16.2	16.2	16.2	16.2	16.1	16.2	16.1	16.2	16.1	16.1	16.1
	16.1	16.1	15.8	16.1	16.1	16.2	16.1	16.1	15.9	16.2	
	16.2	16.1	16.1	16.2	16.2	15.9	16.3	16.2	16.0	16.1	
35°C	20.5	20.4	20.5	20.4	20.4	20.3	20.5	20.1	20.3	20.3	20.3
	20.4	20.4	20.4	20.5	20.3	20.4	20.3	20.3	20.3	20.2	
	20.4	20.3	20.3	20.2	20.3	20.5	20.5	20.2	20.3	20.3	
40°C	23.1	23.0	22.9	23.0	22.9	22.8	22.8	22.7	22.7	22.7	22.8
	23.2	23.1	23.1	22.9	22.9	22.8	22.9	22.7	22.8	22.8	
	23.5	23.2	23.1	23.1	22.9	23.0	22.8	22.9	22.9	22.8	
45°C	27.2	26.9	26.7	26.6	26.5	26.4	26.4	26.3	26.3	26.3	26.3
	26.9	26.7	26.5	26.6	26.4	26.3	26.4	26.2	26.3	26.3	
	26.9	26.8	26.7	26.6	26.6	26.4	26.3	26.2	26.3	26.3	
50°C	31.7	31.2	30.6	30.3	30.3	30.1	30.2	30.1	30.1	30.1	30.2
	30.7	30.4	30.2	30.1	30.2	30.2	30.2	30.1	30.1	30.1	
	31.0	30.7	30.6	30.4	30.2	30.4	30.3	30.3	30.2	30.3	
55°C	39.4	38.8	38.3	37.8	37.3	37.1	36.9	37.0	36.8	36.9	37.0
	39.6	38.9	38.2	37.6	37.0	36.9	37.1	36.9	37.1	37.0	
	40.0	39.2	38.7	38.1	37.5	37.2	37.0	37.1	37.0	37.1	
60°C	50.6	49.8	49.2	48.3	47.6	47.0	46.2	46.2	46.1	46.1	46.2
	51.5	50.4	49.6	49.2	48.4	46.6	46.3	46.2	46.1	46.2	
	49.7	48.3	48.0	47.5	46.9	46.3	46.5	46.4	46.4	46.4	
65°C	60.9	60.6	57.9	56.7	55.2	54.6	54.1	53.9	54	53.9	54.2
	60.5	59.1	58.0	57.3	55.1	54.8	54.5	54.5	54.6	54.4	
	59.6	58.0	57.1	55.8	54.6	54.4	54.3	54.2	54.2	54.3	
70°C	74.7	72.0	69.2	67.7	65.8	64.5	64.3	64.2	64.1	64.2	64.6
	76.6	73.7	72.1	70.3	68.5	68.0	65.9	65.3	64.7	64.6	
	74.0	70.5	69.8	69.0	67.5	66.1	65.1	65.0	64.8	64.9	
75°C	90.0	86.3	82.8	79.7	77.6	77.0	75.5	75.1	75.1	75.0	75.1
	92.8	88.5	86.3	82.8	80.1	78.9	77.6	75.7	75.5	75.3	
	89.4	86.7	84.9	82.3	78.5	76.9	75.9	75.2	75.0	74.9	
80°C	106.3	101.9	98.6	94.8	93.3	90.0	88.5	87.7	87.2	86.7	87.4
	101.0	99.1	96.6	93.0	92.5	90.2	88.8	88.2	88.0	87.9	
	101.7	101.5	97.5	95.7	92.9	89.5	89.2	88.3	87.7	87.4	
85°C	141.2	139.1	133.6	131.5	128.7	125.6	122.8	121.3	118.9	117.6	117.9
	140.5	139.2	136.0	132.7	129.5	128.4	125.9	123.6	120.7	118.3	
	145.6	140.7	139.2	137.1	131.0	130.0	128.3	125.3	121.6	117.8	
90°C	147.7	146.2	144.0	141.0	139.8	137.7	136.2	132.6	130.9	129.4	130.1
	143.1	143.0	140.8	139.2	139.6	137.3	134.7	133.4	131.3	129.9	
	152.6	149.2	145.7	142.9	137.7	135.5	133.6	133.4	132.5	131.1	
95°C	182.6	178.4	175.9	170.4	166.4	159.4	157.7	154.9	153.2	150.9	154.6
	176.2	173.5	170.8	169.3	166.2	164.9	161.3	161.1	159.6	157.6	
	177.6	175.0	171.4	170.7	168.7	169.0	165.6	161.2	158.1	155.2	

Uncertainties were not included in the log graphs below because it was insignificant and allows the best fit line to be clear.



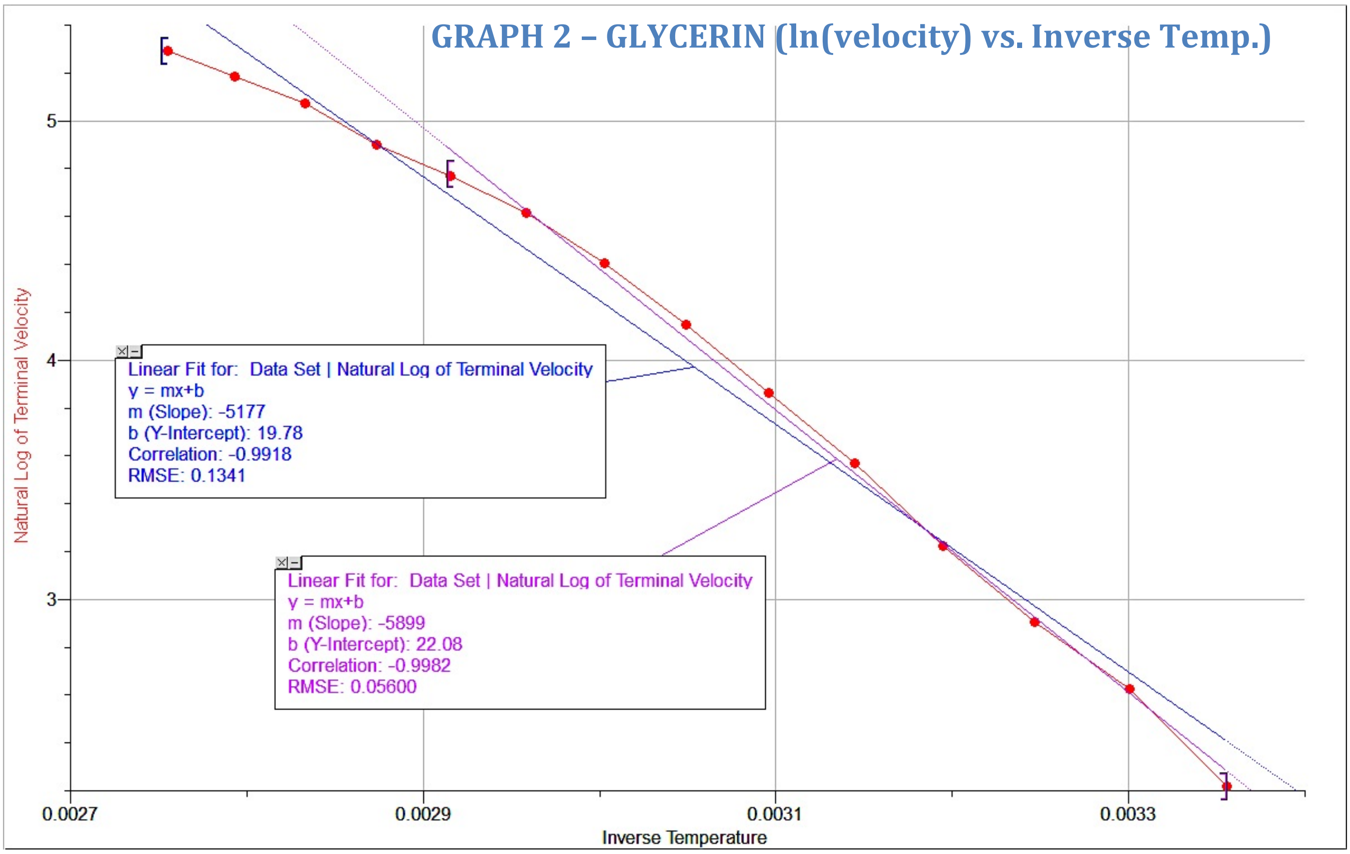
This graph follows the equation $y = -3881x + 15.52$ where $y = \ln(velocity)$ and $x = \frac{1}{T}$. Hence, this relationship can be modeled by $\left[\ln(\text{vel.}) = \frac{-3881}{\text{T}} + 15.52\right]$. The velocity in this case is in $cm{\cdot}s^{-1}$.

This seems to suggest that the hypothesis is correct, as the natural log of velocity plotted against the inverse of temperature, yields in a straight-line graph. At lower temperatures (right side of graph), the data points have an almost constant gradient. However, at higher temperatures, there is greater deviation from the best-fit line. Hence, systematic errors played a role at higher temperatures, as calculations of terminal velocity were not as accurate and terminal velocity was not reached.(See Evaluation for Error Description)

GLYCERIN

Table 5 – Velocity of last 10 frames and Terminal Velocity for Glycerin

Temp.	Last 10 frames before sphere reaches bottom $(cm \cdot s^{-1})$										AVG.
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	
25°C	9.4	9.2	9.6	9.2	9.2	9.2	9.3	9.2	9.2	9.3	**9.2**
	9.2	9.3	9.4	9.2	9.2	9.3	9.2	9.2	9.4	9.2	
	9.3	9.2	9.3	9.2	9.3	9.2	9.4	9.7	9.2	9.2	
30°C	13.9	13.8	13.9	13.9	13.8	13.8	14.1	13.9	13.8	13.8	**13.8**
	13.8	13.9	14	13.8	13.7	14	13.8	13.5	13.5	13.8	
	13.8	13.9	13.9	13.9	13.8	13.9	13.8	13.8	14	13.8	
35°C	18.4	18.3	18.3	18.2	18.3	18.3	18.4	18.3	18.1	18.3	**18.3**
	18.3	18.4	18.3	18.5	18.4	18.3	18.4	18.5	18.3	18.3	
	18.4	18.4	18.3	18.3	18.4	18.3	18.4	18.2	18.3	18.3	
40°C	25.3	25.1	25.1	25.3	25.2	25.1	25.2	25.1	25.1	25.1	**25.1**
	25.2	25.1	25.2	25.2	25.3	25.3	25.1	25.1	25.1	25.2	
	25.3	25.2	25.2	25.4	25.2	25.3	25.3	25.1	25.2	25.1	
45°C	35.5	35.5	35.6	35.8	35.6	35.7	35.4	35.5	35.5	35.5	**35.5**
	35.6	35.5	35.6	35.6	35.6	35.4	35.5	35.8	35.6	35.5	
	35.4	35.6	35.7	35.5	35.6	35.7	35.7	35.6	35.4	35.5	
50°C	48.1	48	47.9	47.7	47.7	47.6	47.5	47.5	47.6	47.5	**47.6**
	47.9	47.9	47.7	47.5	47.7	47.6	47.5	47.5	47.5	47.6	
	48.1	48	48.1	47.8	47.7	47.6	47.7	47.7	47.6	47.7	
55°C	63.9	63.8	63.6	63.7	63.4	63.2	63.2	63.2	63.3	63.2	**63.2**
	64.5	64.1	63.8	63.5	63.3	63.1	63.2	63.1	63.1	63.1	
	64.1	64	63.7	63.5	63.2	63.1	63.3	63.2	63.3	63.2	
60°C	83.4	83	82.6	82.4	82.1	81.9	81.7	81.5	81.5	81.5	**81.7**
	82.5	82.2	81.9	81.7	81.8	81.9	81.8	81.7	81.8	81.8	
	83.1	82.8	82.5	82.2	82.1	81.8	81.7	81.8	81.9	81.8	
65°C	102.9	102.4	102.1	101.9	101.7	101.6	101.4	101.5	101.5	101.5	**101.2**
	104	103.5	103.1	103	102.4	101.9	101.9	101.4	101.3	101.3	
	103.1	103	102.3	101.9	101.6	101.2	101	100.8	100.9	100.9	
70°C	120.1	119.6	119.2	118.9	118.4	118	117.8	117.6	117.5	117.6	**118.0**
	119.7	119.5	119.3	118.9	118.8	118.8	118.4	118.2	118.2	118.2	
	120.3	119.8	119.3	119.2	119	118.7	118.3	118.1	118	118	
75°C	137.7	137.2	136.7	136.4	135.9	135.3	134.9	134.4	134.1	134	**134.0**
	138.4	137.7	137.2	136.6	136	135.9	135	134.5	134.2	134.1	
	137.2	136.8	136.4	136.1	135.6	135.1	134.8	134.4	134.1	134	
80°C	165.8	165.2	164.3	163.7	162.7	162	161.1	160.8	159.8	158.9	**159.7**
	167.2	166.2	165.4	164.6	163.9	163	162.3	161.7	161.7	161.2	
	169.2	167	165.4	163.8	162.3	161.7	160.4	159.8	159.3	158.9	
85°C	186.5	184.1	182.8	181.1	180.4	179.2	179.1	178.5	177.6	176.8	**178.2**
	189.3	187.8	186	184.9	183.3	181.9	181	180	179.1	178.1	
	187.5	186.1	184.9	183.1	182.3	181.7	180.9	180.2	179.7	179.6	
90°C	226.6	223.5	220.9	218.3	215.8	213.1	211.2	208.4	206.7	204.3	**198.5**
	212.6	210.4	208.5	206.1	205.6	202.8	201.2	199.7	198.5	196.7	
	214.1	211.3	209.5	206.9	204.9	202.4	201.3	198.9	196.5	194.4	



The blue line represents the linear fit for all the data points. However, this best line of fit resulted in significant deviation at low and high temperatures. On visual analysis, this is a result of the sharp decrease in gradient that occurs at 70°C. Hence another best-fit line (pink) was constructed from $25 - 70$°C.

The pink best-fit line accurately defines the relationship until 70°C. This suggests that at temperatures above that, the data was slightly skewed, due to a number of both random and systematic errors (See Evaluation).

This graph also seems to suggest that the second part of my hypothesis was correct, as this yields in a straight-line graph.

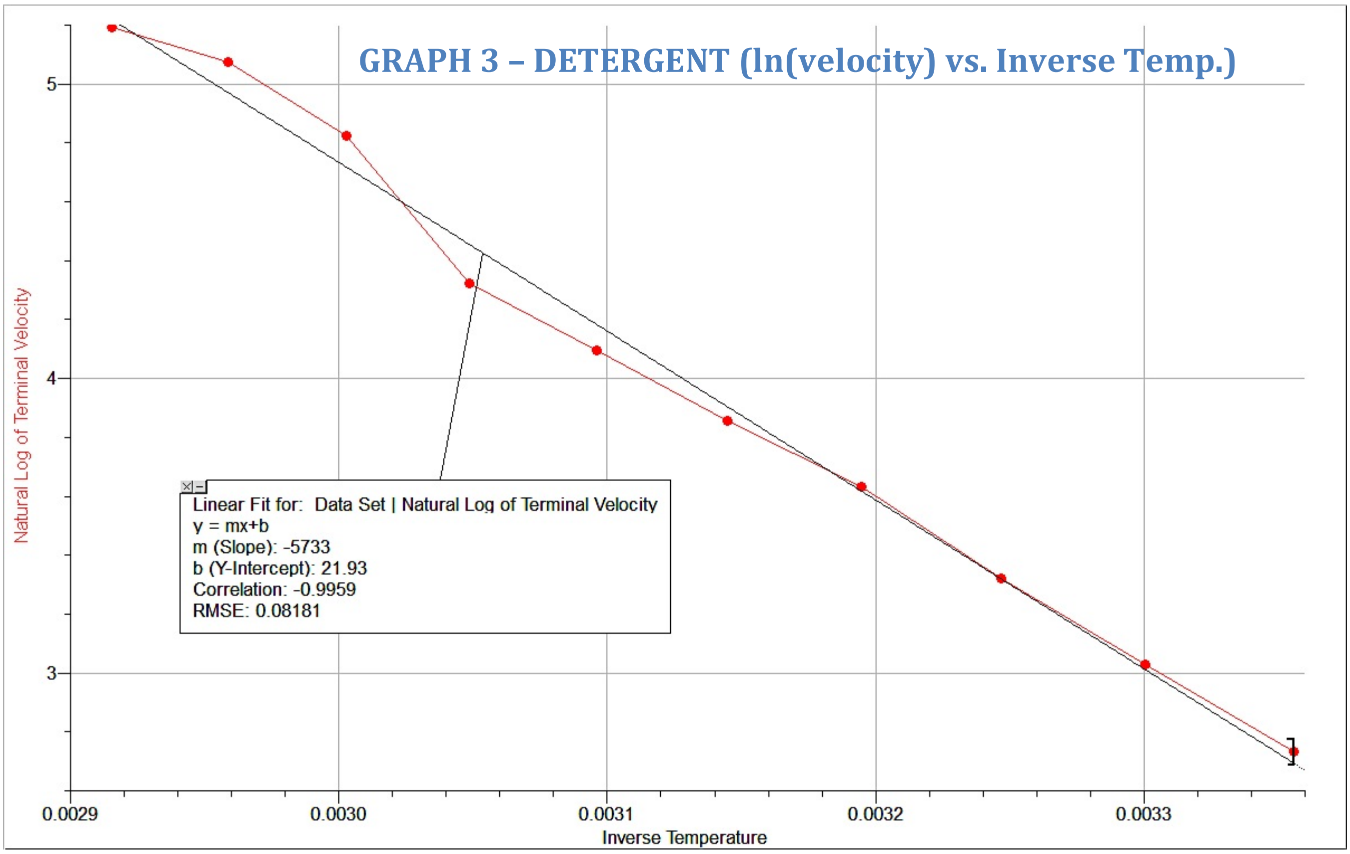
The pink best fit line will be considered against the limit $25 - 70$°C. This graph follows the equation $y = -5899x + 22.08$ where $y = \ln(velocity)$ and $x = \frac{1}{T}$. Hence, $\left[\ln(\text{vel.}\ in\ \text{cm·s}^{-1}) = \frac{-5899}{\text{T}} + 22.08\right]$.

Precision is also lost as temperatures increase as can be seen visually in the table. At higher temperatures, terminal velocity varies greatly over the three trials.

DISHWASHING SOAP/DETERGENT

Table 6 – Velocity of last 10 frames and Terminal Velocity for Detergent

Temp.	**Last 10 frames before sphere reaches bottom ($cm \cdot s^{-1}$)**										**AVG.**
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	
25°C	15.3	15.4	15.4	15.2	15.3	15.3	15.4	15.5	15.3	15.3	**15.4**
	15.5	15.4	15.3	15.3	15.3	15.5	15.3	15.4	15.3	15.4	
	15.3	15.3	15.5	15.3	15.4	15.2	15.3	15.4	15.3	15.5	
30°C	20.7	20.6	20.8	20.6	20.6	20.6	20.6	20.6	20.6	20.6	**20.7**
	20.7	20.6	20.8	20.6	20.6	20.8	20.8	20.6	20.6	20.6	
	20.9	20.8	20.8	20.7	20.6	20.7	20.9	20.8	20.7	20.7	
35°C	27.8	27.6	27.6	27.8	27.9	27.7	27.8	27.7	27.7	27.6	**27.7**
	27.6	27.8	27.7	27.6	27.6	27.6	27.6	27.6	27.6	27.6	
	27.7	27.6	27.8	27.6	27.7	27.6	27.8	27.7	27.7	27.7	
40°C	38.3	38.2	38.0	38.0	37.9	37.9	37.8	37.7	37.9	37.8	**37.8**
	38.4	38.2	38.3	38.1	38.0	37.9	37.9	37.8	37.8	37.8	
	38.2	38.2	38.0	38.1	38.0	38.0	37.9	37.9	37.9	37.8	
45°C	50.1	49.6	49.1	48.5	48.0	47.2	47.3	47.1	47.2	47.1	**47.3**
	49.7	49.2	48.8	48.3	47.8	47.5	47.3	47.3	47.3	47.3	
	52.4	50.3	49.9	48.7	48.0	47.5	47.4	47.6	47.4	47.4	
50°C	70.4	68.2	65.9	64.6	63.2	61.9	60.9	60.3	60.2	60.2	**60.0**
	69.8	67.1	65.9	63.1	61.9	60.4	59.9	60.0	59.8	59.9	
	68.4	66.7	64.0	63.0	61.8	61.0	60.0	59.9	59.7	59.7	
55°C	90.4	87.3	84.0	82.1	79.7	78.3	76.9	76.0	75.4	75.3	**75.3**
	85.3	82.8	80.1	78.7	76.5	76.2	75.8	75.7	75.9	75.7	
	86.2	86.1	81.9	79.4	78.1	76.6	75.0	74.7	74.8	74.8	
60°C	144.4	142.6	140.0	138.7	135.7	132.8	129.5	127.6	126.0	124.9	**124.7**
	140.3	140.1	137.6	135.9	133.7	130.9	129.1	127.5	126.5	125.3	
	142.2	141.0	139.7	136.4	134.5	131.8	129.6	126.6	123.9	123.9	
65°C			180.7	176.7	173.5	169.8	167.0	164.2	162.3	160.2	**160.1**
			174.6	172.1	170.6	167.4	165.5	163.9	160.0	158.4	
			176.3	174.2	171.8	168.7	166.3	166.2	163.8	161.7	
70°C					196.4	192.5	189.5	186.2	183.2	181.7	**180.1**
					193.3	190.6	187.9	184.1	181.0	179.4	
					196.1	191.4	187.6	184.3	181.9	179.2	

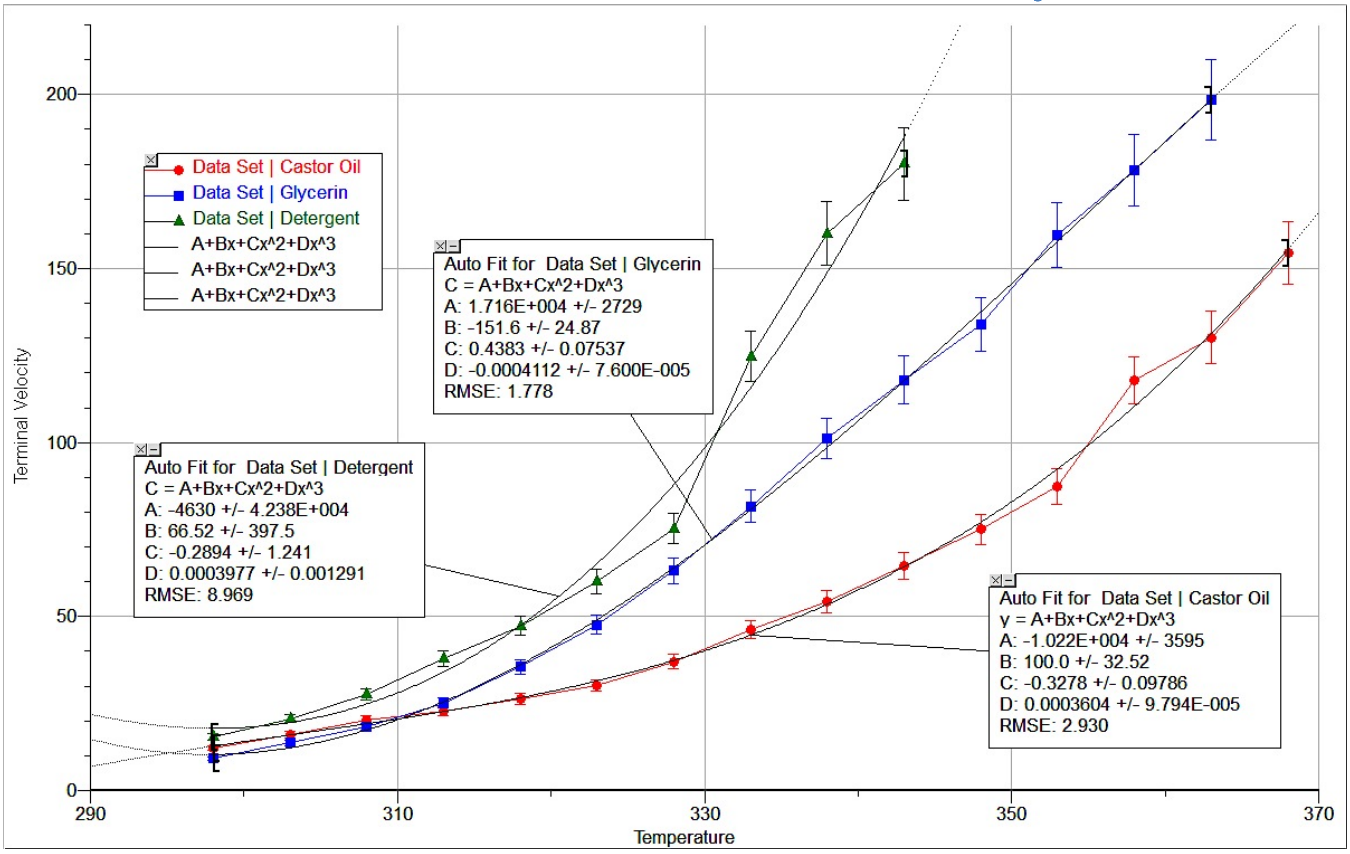


At and after 65°C, the liquid began to become clouded and started changing colour. Additionally, the liquid could not exceed temperatures of 70°C, above which it started to rise and evaporate. Even for 65°C and 70°C, the clouded liquid would collect at the top. Hence, the sphere could not be distinguished for the first few frames. Additional uncertainty caused due to difference in density between top of tube and bottom of tube. (See Evaluation)

This graph follows the equation $y = -5733x + 21.93$ where $y = \ln(velocity)$ and $x = \frac{1}{T}$. Hence, $\left[\ln(\text{vel.}) = \frac{-5733}{\text{T}} + 21.93\right]$. The velocity in this case is in cm·s^{-1}.

This graph also suggests that the second part of the hypothesis is true, since these parameters resulted in a straight-line graph.

GRAPH 4: TERMINAL VELOCITY vs. TEMPERATURE FOR 3 LIQUIDS



Surprisingly, when plotted, the graphs are similar in shape. When curve fitted, all the best-fit curves take the form of $y = x^3$, cubic curves. However, the specific cubic curves found (produced below) will only prove true for the specific dimensions of sphere and densities of liquid used in this experiment and a limit of temperatures.

Castor Oil – $v(T) = (3.6 \cdot 10^{-4})T^3 - 0.32T^2 + 100T - (1.02 \cdot 10^4)$
Glycerin – $v(T) = -(4.1 \cdot 10^{-4})T^3 + 0.44T^2 - 152T + (1.72 \cdot 10^4)$
Liquid Detergent – $v(T) = (4.0 \cdot 10^{-4})T^3 - 0.30T^2 + 67T - 4630$

It can hence be generalized that for these three liquids and probably most liquids, over the range of temperatures of 25°C − 95°C, the relationship of velocity against temperature can be approximated by x^3 curves.

7.3 Viscosity Analysis for 3 liquids

The viscosity of the liquids is calculated using **Equation 7** reproduced below and the data tables above (average terminal velocities). A java program (*Appendix 2*) was used to compute this equation. Spreadsheet software could have also been used, however, would become complex when calculating percentage error.

$$\eta = \frac{2r^2 \cdot g \cdot (\rho_{sphere} - \sigma_{liquid})}{9v}$$

Radius of Sphere: 0.0026125 m $\pm$ 0.096% (Table 1)
Density of Sphere: 6025 kg·m^{-3} $\pm$ 0.510% (Table 1)
Acceleration due to gravity: 9.81 m·s^{-2}

Calculation of Density

$$\text{Density} = \frac{\text{Mass}}{\text{Volume}}$$

A measuring cylinder of 10cm^3 was used for all volume calculations. (10 $\pm$ 0.1) cm^3 was the volume of liquid used, and the mass was calculated on an electronic mass balance.

Table 7 – Measure of Density of Liquids

Liquid	Mass Before (grams) $\pm$0.01	Mass After (grams) $\pm$0.01	Change in Mass (g) $\pm$0.02	Volume (cm^3) $\pm$0.1	**Density kg·m^{-3} $\approx$ 1.2%**
Castor Oil	30.36	39.93	9.57	10	**957**
Glycerin	30.54	43.16	12.62	10	**1262**
Detergent	30.35	40.56	10.21	10	**1021**

Density of Castor Oil: 957 kg·m^{-3} $\pm$ 1.2%
Density of Glycerin: 1262 kg·m^{-3} $\pm$ 1.2%
Density of Liquid Detergent: 1021 kg·m^{-3} $\pm$ 1.2%

Terminal Velocity values are taken from the "average" column from tables above. All units are converted to SI units. Hence, viscosity has the unit: **Pa·s**

The viscosity for each temperature for each liquid was calculated using the equation above and a Java program. The data is summarized below.

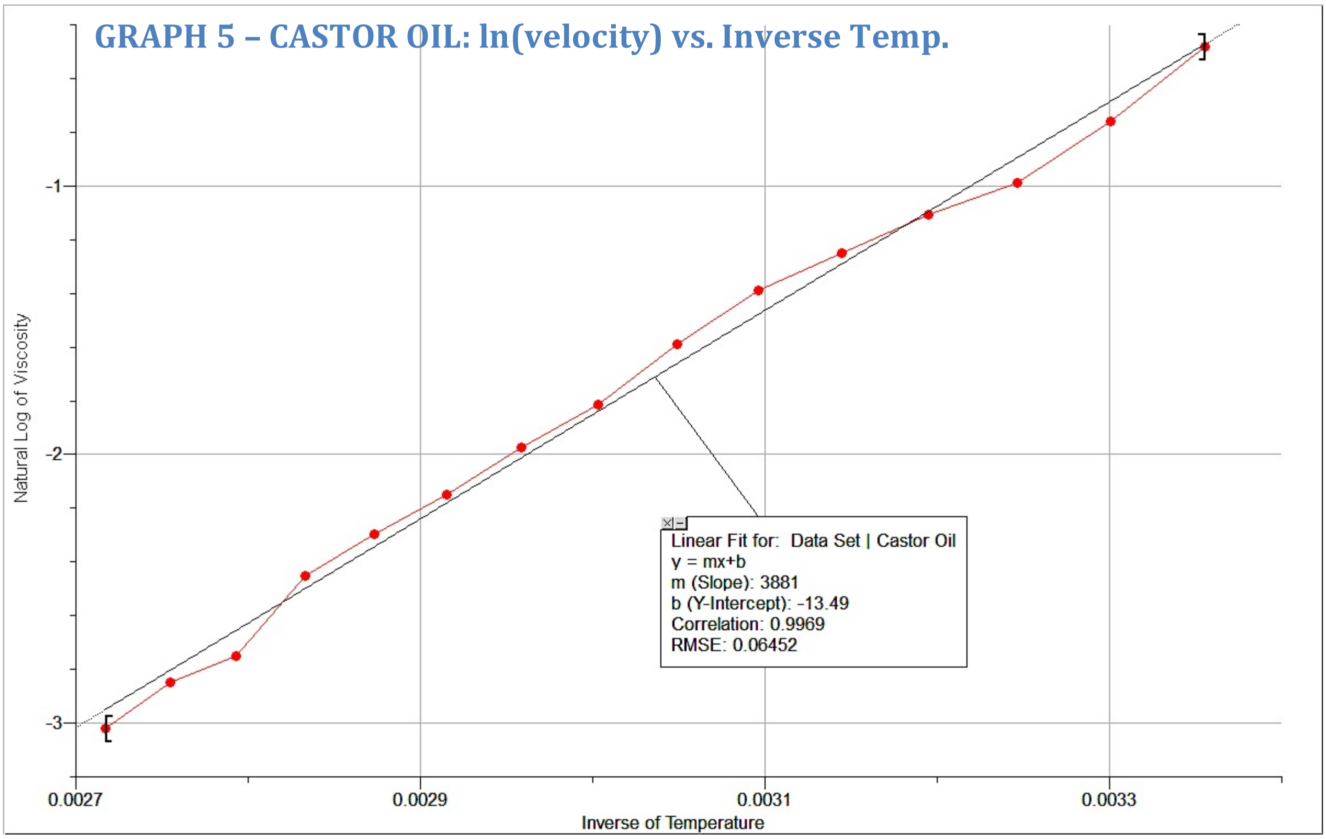
Calculation of Viscosity

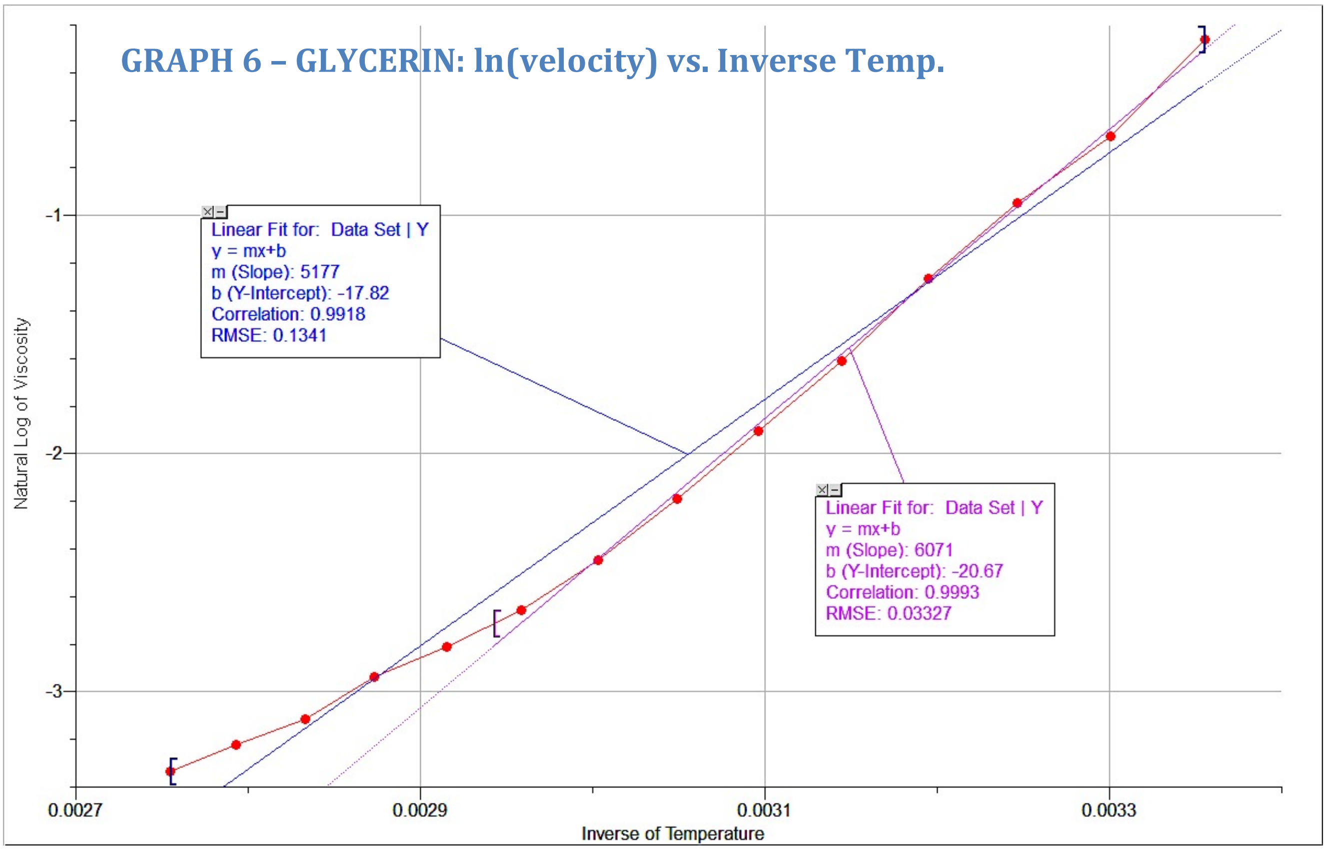
Table 8 – Statement of Viscosity of Liquids

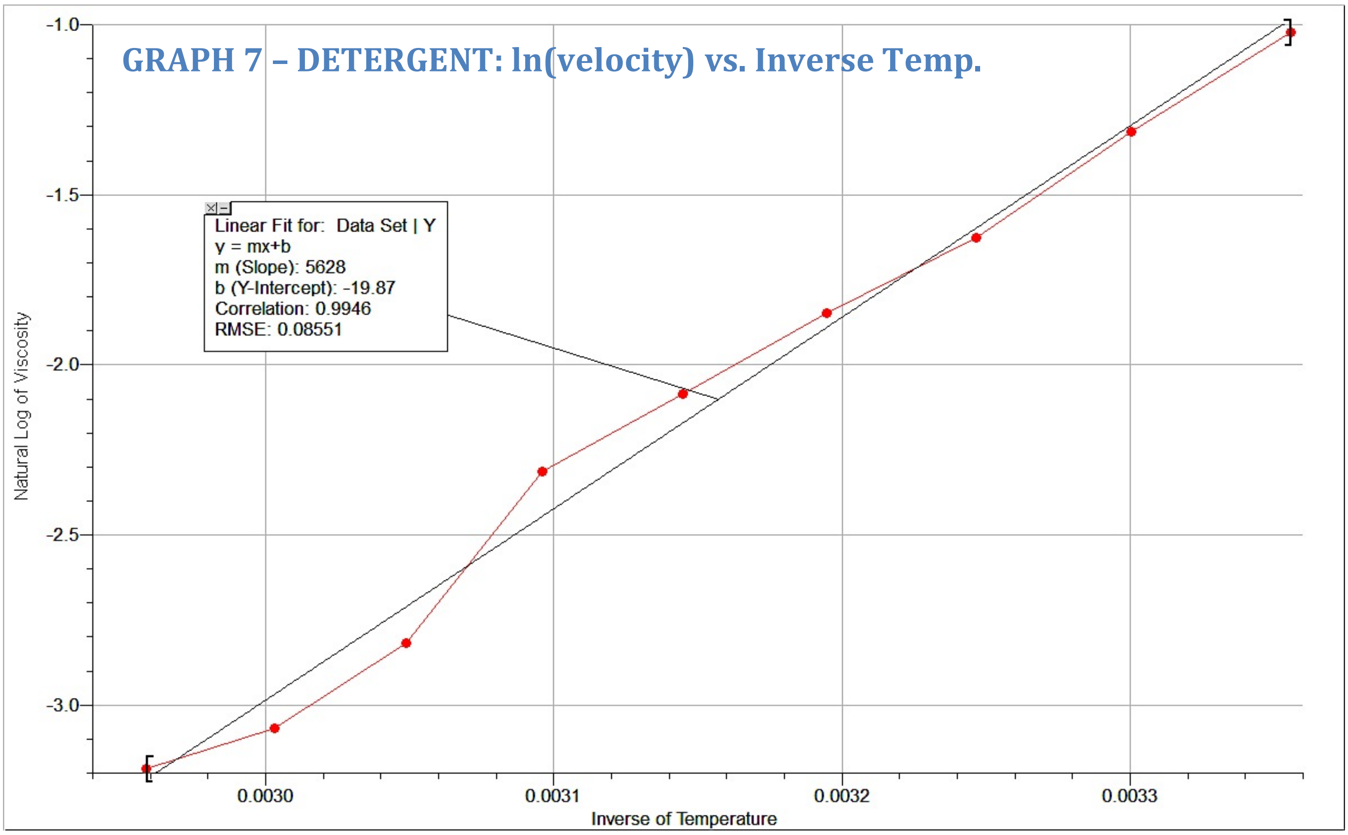
Temp.	Castor Oil		Glycerin		Liquid Det.	
	Term. Vel. ($cm \cdot s^{-1}$)	**Viscosity (Pa·s) (10^{-1})**	Term. Vel. ($cm \cdot s^{-1}$)	**Viscosity (Pa·s) (10^{-1})**	Term. Vel. ($cm \cdot s^{-1}$)	**Viscosity (Pa·s) (10^{-1})**
25°C	12.2	**6.18**	9.2	**7.70**	15.4	**4.84**
30°C	16.1	**4.68**	13.8	**5.14**	20.7	**3.60**
35°C	20.3	**3.72**	18.3	**3.87**	27.7	**2.69**
40°C	22.8	**3.31**	25.1	**2.82**	37.8	**1.97**
45°C	26.3	**2.87**	35.5	**2.00**	47.3	**1.57**
50°C	30.2	**2.50**	47.6	**1.49**	60.0	**1.24**
55°C	37.0	**2.04**	63.2	**1.12**	75.3	**0.989**
60°C	46.2	**1.63**	81.7	**0.867**	124.7	**0.597**
65°C	54.2	**1.39**	101.2	**0.700**	160.1	**0.465**
70°C	64.6	**1.17**	118.0	**0.601**	180.1	**0.413**
75°C	75.1	**1.00**	134.0	**0.529**		
80°C	87.4	**0.863**	159.7	**0.444**		
85°C	117.9	**0.640**	178.2	**0.398**		
90°C	130.1	**0.580**	198.5	**0.357**		
95°C	154.6	**0.488**				

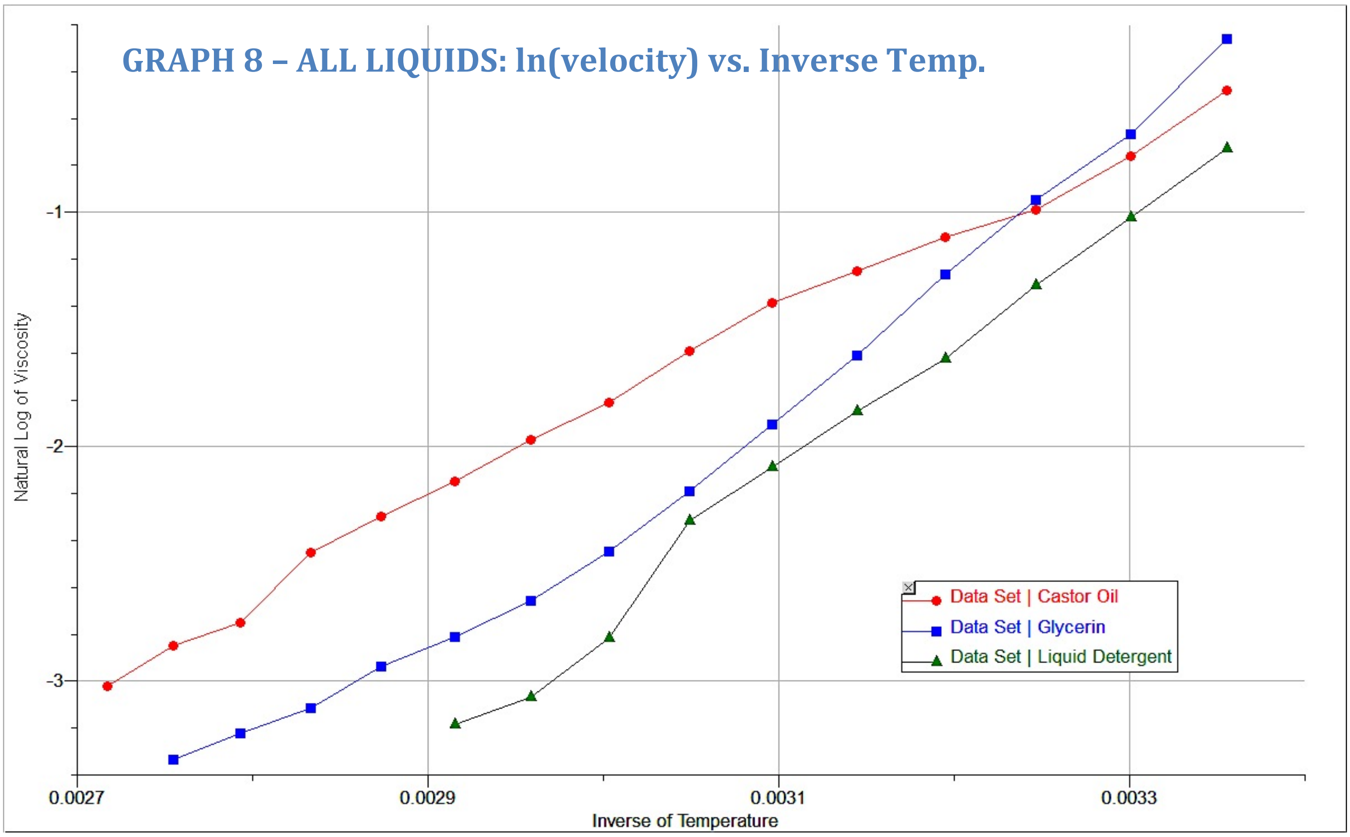
Based on the data above, graphs are made below of the natural of log against the inverse of temperature, as the hypothesis suggests. The graphs were plotted for each liquid, in order to analyze the line of best fit, and formulate a relationship based on it. Then all three liquids were plotted on the same graph to analyze the similarities and differences between the three.

The combined graphs also allowed the evaluation of accuracy, and comparison of the three liquids in general.


GRAPH 5 – CASTOR OIL: ln(velocity) vs. Inverse Temp.
Linear Fit for: Data Set | Castor Oil
y = mx+b
m (Slope): 3881
b (Y-Intercept): -13.49
Correlation: 0.9969
RMSE: 0.06452
Natural Log of Viscosity
-1
-2
-3
0.0027
0.0029
0.0031
0.0033
Inverse of Temperature


GRAPH 6 – GLYCERIN: ln(velocity) vs. Inverse Temp.
Linear Fit for: Data Set | Y
y = mx+b
m (Slope): 5177
b (Y-Intercept): -17.82
Correlation: 0.9918
RMSE: 0.1341
Linear Fit for: Data Set | Y
y = mx+b
m (Slope): 6071
b (Y-Intercept): -20.67
Correlation: 0.9993
RMSE: 0.03327
Natural Log of Viscosity
-1
-2
-3
0.0027
0.0029
0.0031
0.0033
Inverse of Temperature


GRAPH 7 – DETERGENT: ln(velocity) vs. Inverse Temp.
Linear Fit for: Data Set | Y
y = mx+b
m (Slope): 5628
b (Y-Intercept): -19.87
Correlation: 0.9946
RMSE: 0.08551
Natural Log of Viscosity
Inverse of Temperature
-1.0
-1.5
-2.0
-2.5
-3.0
0.0030
0.0031
0.0032
0.0033


GRAPH 8 – ALL LIQUIDS: ln(velocity) vs. Inverse Temp.
Data Set | Castor Oil
Data Set | Glycerin
Data Set | Liquid Detergent
Natural Log of Viscosity
Inverse of Temperature
-1
-2
-3
0.0027
0.0029
0.0031
0.0033

The graphs above are similar to the hypothesis stated. Summarized below –
Since $y = \ln(\text{visc.})$ and $x = \frac{1}{T}$, the equations of the line of best fit are -

Castor Oil → $\ln(\text{visc.}) = \frac{3881}{T} - 13.49 \ \lim_T[298,368]$ K

Glycerin → $\ln(\text{visc.}) = \frac{6071}{T} - 20.67 \ \lim_T[298,343]$K

Castor Oil → $\ln(\text{visc.}) = \frac{5628}{T} - 19.87 \ \lim_T[298,343]$ K

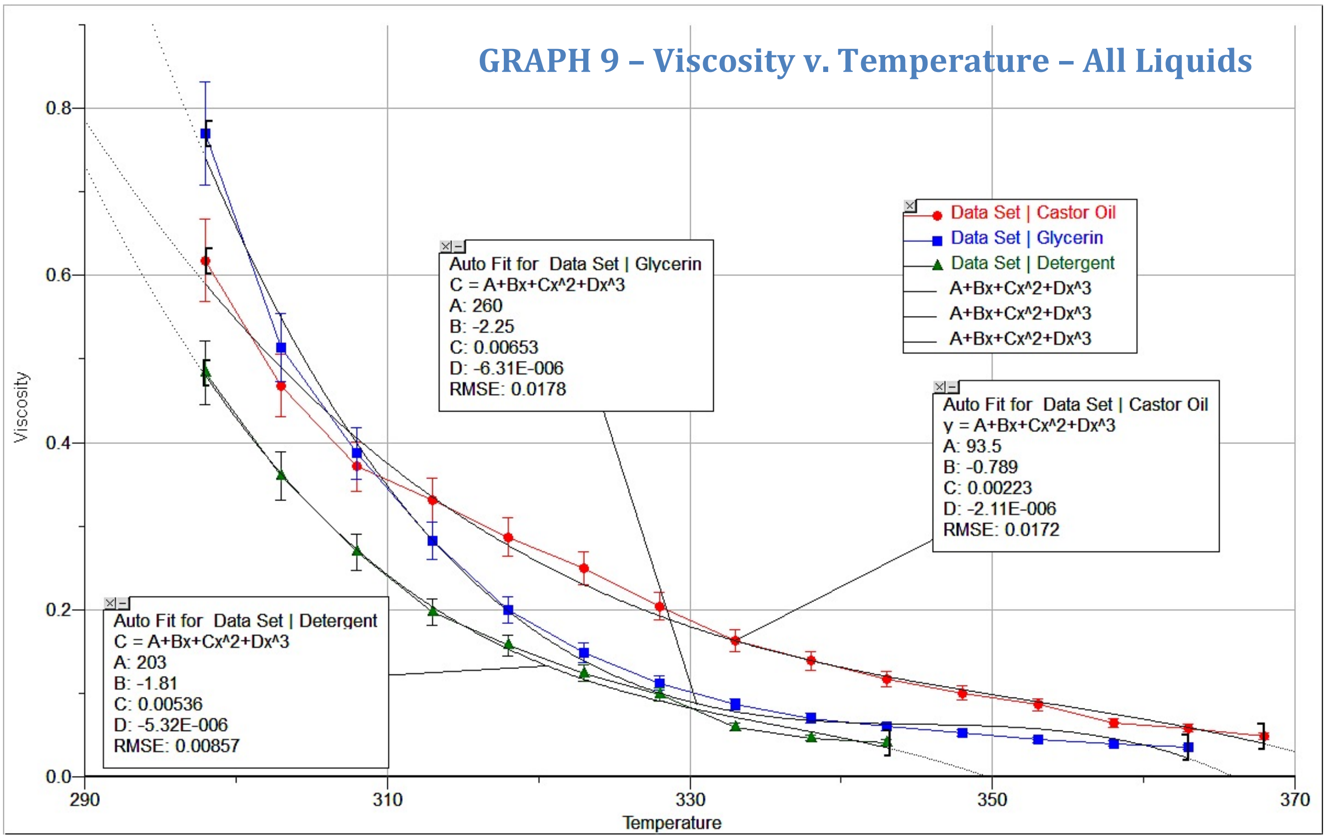
The line of best fit above can model the viscosity of the liquids against absolute temperature, for the limits stated.

The Graphs above were quite accurate and precise in following this trend. Graphs 5 and 7 (Castor Oil and Liquid Detergent) seemed to follow this trend for all temperatures at which viscosity was measured. However Graph 5 (Castor Oil) shows less deviation. The viscosity of liquid detergent deviated significantly from 40°C. This is due to the fact that the viscosity of liquid detergent is lesser than that of castor oil and has greater error. (see Evaluation).

Hence, it could be concluded that for these two liquids, had errors been further reduced and eliminated, all data points of the viscosity observed would adhere more closely to the relationship.

The graph for Glycerin (Graph 6) is unlike the other two. Even though the viscosities of glycerin were experimentally calculated until 95°C, the graph seems to follow one relationship until 70°C, and another after that. Until 70°C, the data points adhere very closely to the relationship calculated by the pink line of best fit.

Finally this error or anomaly in glycerin is supported by the combined graph (Graph 8). Until 35°C, the viscosity of glycerin is always higher than that of castor oil. However after this, it mysteriously dips below that of castor oil. This could be caused due to impure chemicals.



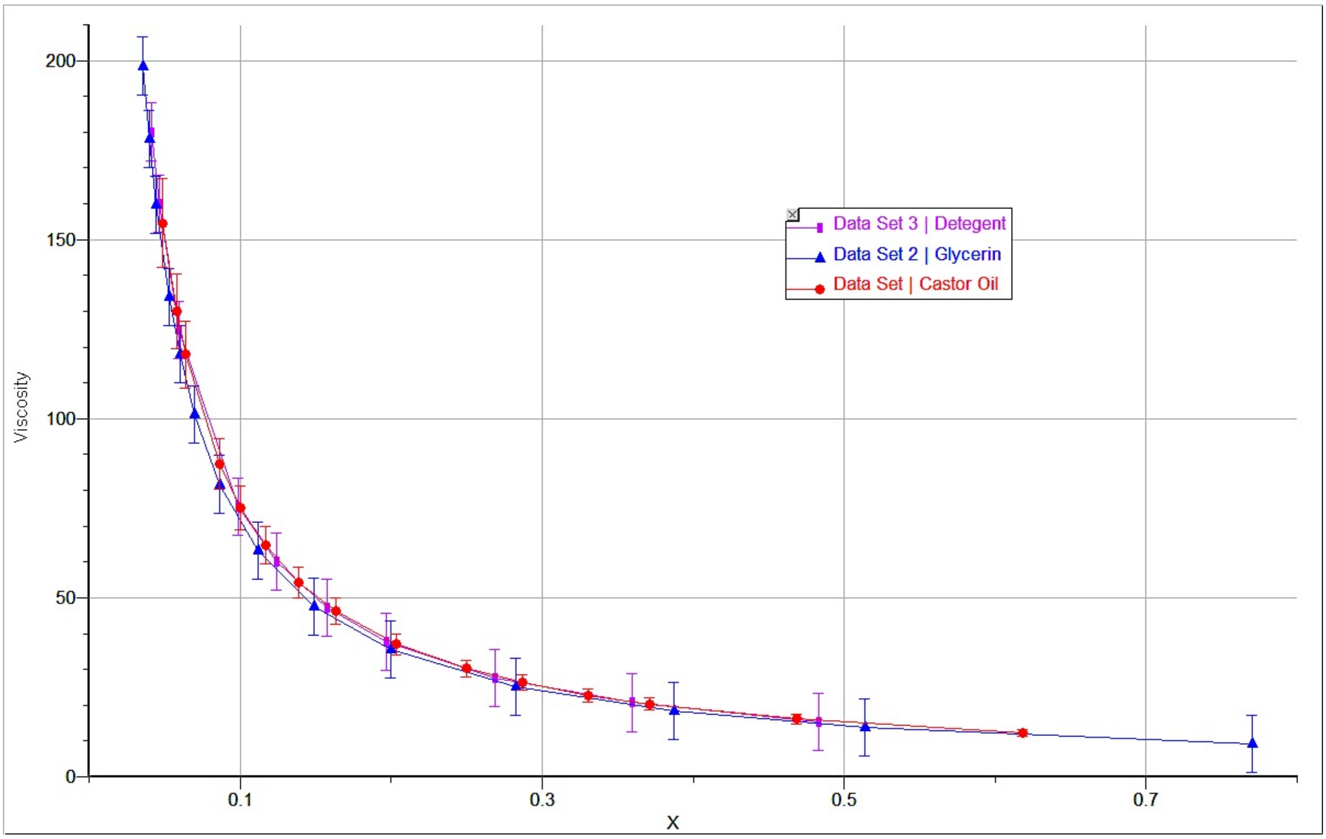
This graph, similar to the direct Terminal Velocity v. Temperature graph, also fits cubic equations. From this analysis, it can be concluded that viscosities decrease with respect to temperature, cubically. However, the specific cubic curves found (produced below) will only prove true for the specific densities and temperatures used in this investigation.

Castor Oil – $\eta(T) = -(2.1 \cdot 10^{-6})T^3 - 0.002T^2 - 0.79T + 93.5$
Glycerin – $\eta(T) = -(6.3 \cdot 10^{-6})T^3 + 0.006T^2 - 2.25T + 260$
Detergent – $\eta(T) = -(5.32 \cdot 10^{-6})T^3 - 0.005T^2 - 1.81T + 203$

Hence, it can also be generalized that for similar liquids too, the viscosities can be estimated by cubic graphs against temperature. However, such a generalization is just an estimation and does not reflect exact data points. (As seen in the graph, the curve fitting does not pass through every data point found experimentally)

This is the main conclusion from this investigation as viscosity values can be directly estimated at specific temperatures with these equations.

GRAPH 10 – Viscosity v. Terminal Velocity – All Liquids



This graph is one of the most interesting graphs as it shows that for all three liquids, the viscosity v. terminal velocity lie on the same line. This does not mean that liquids have the same viscosity, but rather, when they do have the same viscosity (at different temperatures), the terminal velocity of the sphere used in this investigation will also be the same.

$$\eta = \frac{2r^2 \cdot g \cdot (\rho_{sphere} - \sigma_{liquid})}{9v}$$

Even though the equation for viscosity (reproduced above) does feature terminal velocity, it also features other factors that are not constant for the three liquids, which is the density of the liquids. All three liquids have different densities, which should transform the viscosity-velocity curve, however, as shown by the graph, it doesn't have the effect expected. Hence, for estimation purposes, other factors can be ignored. However, the densities of the liquids were similar in this investigation. Perhaps, for widely varying densities, there would be greater difference.

7.4 Terminal Velocity and Reynolds's Number

$$Re = \frac{2r \cdot velocity \cdot \sigma_{liquid}}{\eta}$$

Where η is the viscosity and r is the radius of the sphere.
The terminal velocities are taken from above and Reynolds's number calculated for each of the terminal velocities to evaluate reliability of results. To calculate Reynolds's number, I made a Java Program to compute it automatically (Appendix 3).

Table 9 – Calculation of Reynolds Number

Temp	Ter. Vel.	Visc.	Rey. No.	Ter. Vel.	Visc.	Rey. No.	Ter. Vel.	Visc.	Rey. No.
	CASTOR OIL			GLYCERIN			DETERGENT		
25°C	12.2	6.18	**0**	9.2	7.70	**0**	15.4	4.84	**0**
30°C	16.1	4.68	**0**	13.8	5.14	**0**	20.7	3.60	**0**
35°C	20.3	3.72	**0**	18.3	3.87	**0**	27.7	2.69	**0**
40°C	22.8	3.31	**0**	25.1	2.82	**0**	37.8	1.97	**0**
45°C	26.3	2.87	**0**	35.5	2.00	**0**	47.3	1.57	**0**
50°C	30.2	2.50	**0**	47.6	1.49	**0**	60	1.24	**0**
55°C	37.0	2.04	**0**	63.2	1.12	**0**	75.3	0.989	**1**
60°C	46.2	1.63	**0**	81.7	0.867	**1**	124.7	0.597	**2**
65°C	54.2	1.39	**1**	101.2	0.700	**1**	160.1	0.465	**3**
70°C	64.6	1.17	**1**	118.0	0.601	**1**	180.1	0.413	**4**
75°C	75.1	1.00	**1**	134.0	0.529	**2**			
80°C	87.4	0.863	**2**	159.7	0.444	**3**			
85°C	117.9	0.640	**3**	178.2	0.398	**4**			
90°C	130.1	0.580	**3**	198.5	0.357	**5**			
95°C	154.6	0.488	**5**						

The Reynolds number values above are estimated to one significant number because as long as the value is less than or equal to one, the flow is almost perfectly laminar. However, as it increases above 1, turbulent forces start playing a role in the movement of the sphere through the liquid.

From these values, it can be concluded that for Castor Oil, turbulent forces start at 75°C, 70°C for glycerin and 55°C for liquid detergent. Hence, part of uncertainty and deviation after these temperatures is expected to be due to the fact of other forces (turbulent forces) acting on the sphere randomly.

8. Conclusion

The aim of this research was to evaluate the effect of temperature on the viscosity of the liquid. This research not only provided conclusive proofs to the mathematical hypothesis, and hence formed a viable relationship between viscosity and temperature; other surprising conclusions were drawn from the graphs plotted.

Terminal Velocity – Temperature relationship
The relationship between terminal velocity and temperature describes in the hypothesis was confirmed to some extent. A graph plotting the natural log of velocity and the inverse of temperature, did yield in a straight line graph with the equations of best fit –

1. Castor Oil → $\ln(\text{vel.}) = \frac{-3881}{\text{T}} + 15.52\ \lim_T[298,368]\ \text{K}$
2. Glycerin → $\ln(\text{vel.}) = \frac{-5899}{\text{T}} + 22.08\ \lim_T[298,343]\text{K}$
3. Liquid Detergent → $\ln(\text{vel.}) = \frac{-5733}{\text{T}} + 21.93\ \lim_T[298,343]\text{K}$

As mentioned in the hypothesis, the theoretical model used was $\ln(v) = \ln\left(\frac{b}{a}\right) - \frac{E_a}{RT}$.
This model was proved to some extent, as graphs of natural log and inverse temperature lead to straight-line graphs.

From the experimentally measured terminal velocities (Graph 4), it was found that the terminal velocity achieved could be related to the temperature by modeling it to a cubic curve.

Castor Oil – $v(T) = (3.6 \cdot 10^{-4})T^3 - 0.32T^2 + 100T - (1.02 \cdot 10^4)$
Glycerin – $v(T) = -(4.1 \cdot 10^{-4})T^3 + 0.44T^2 - 152T + (1.72 \cdot 10^4)$
Detergent – $v(T) = (4.0 \cdot 10^{-4})T^3 - 0.30T^2 + 67T - 4630$

This hints that the terminal velocity of similarly "thick" liquids can also be modeled and estimated by cubic curves.

Viscosity – Temperature relationship

Hence, after measuring the velocity, the viscosity of the liquids at those temperature were calculated using **Equation 7** (Stokes' Law).

This relationship supported the theoretical hypothesis, across a limit of temperatures that followed the relationship showed by the line of best fit.

Castor Oil → $\ln(\text{visc.}) = \frac{3881}{T} - 13.49\ \lim_T[298,368]\ \text{K}$
Glycerin → $\ln(\text{visc.}) = \frac{6071}{T} - 20.67\ \lim_T[298,343]\text{K}$
Castor Oil → $\ln(\text{visc.}) = \frac{5628}{T} - 19.87\ \lim_T[298,343]\ \text{K}$

The equations above can be compared to the model proposed in the hypothesis:
$\ln(\eta) = \ln(a) + \frac{E_a}{RT}$, and hence proved.

It was also found that viscosity decreases cubically with absolute temperature. Using the equations reproduced below, viscosity can be estimated using absolute temperature.

Castor Oil – $\eta(T) = -(2.1 \cdot 10^{-6})T^3 - 0.002T^2 - 0.79T + 93.5$
Glycerin – $\eta(T) = -(6.3 \cdot 10^{-6})T^3 + 0.006T^2 - 2.25T + 260$
Detergent – $\eta(T) = -(5.32 \cdot 10^{-6})T^3 - 0.005T^2 - 1.81T + 203$

This is the most important conclusion as it allows us to estimate directly the values of viscosity. From a commercial perspective regarding engine oils, if such equations can be found, then behavior of those oils can be predicted at high temperatures.

Viscosity – Terminal Velocity relationship

Viscosity and terminal velocity vary inversely as the equation suggests. However, what was surprisingly concluded was that the difference in densities of the liquid, in this investigation, did not make any significant difference.

9. Evaluation

The experiment was reliably conducted, as the result approximately follows the mathematical relationship, and equations could be generated for each liquid. The aim was realized. The rate of heat loss was very slow as the experiment container was plastic and additional insulation was used. This confirmed that all trials were conducted in congruent situations.

The log graph created above surprisingly follows the hypothesis to a great extent. This could be due to the fact that a log graph seems to minimize error. The data points adhere to the best fit line, however deviate significantly at higher temperatures.

Additionally, because a micrometer, electronic mass balance and computer software was used to calculate data, the uncertainty was minimized. The greatest uncertainty was caused when calculating density, as the most accurate measure for volume that was used was a graduated cylinder.

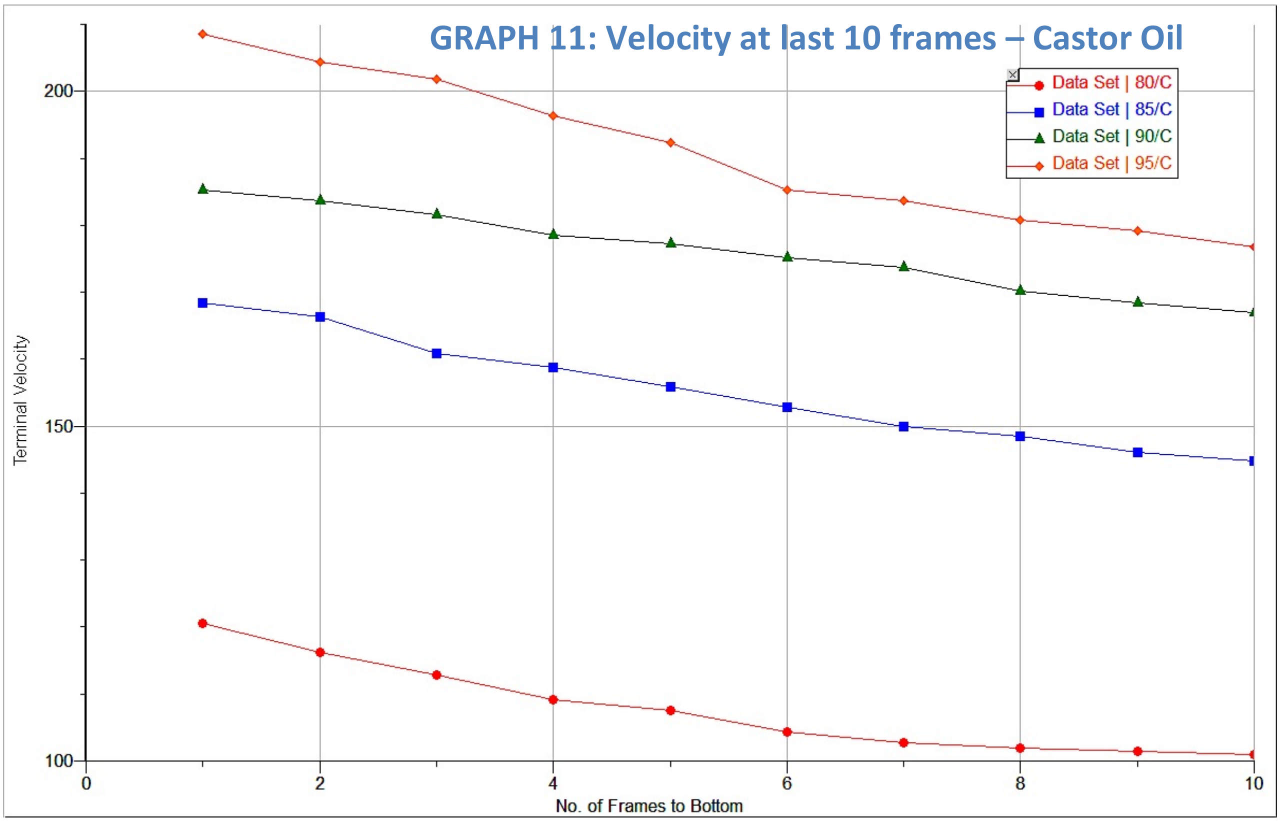
Random Error: Blurring on Camera

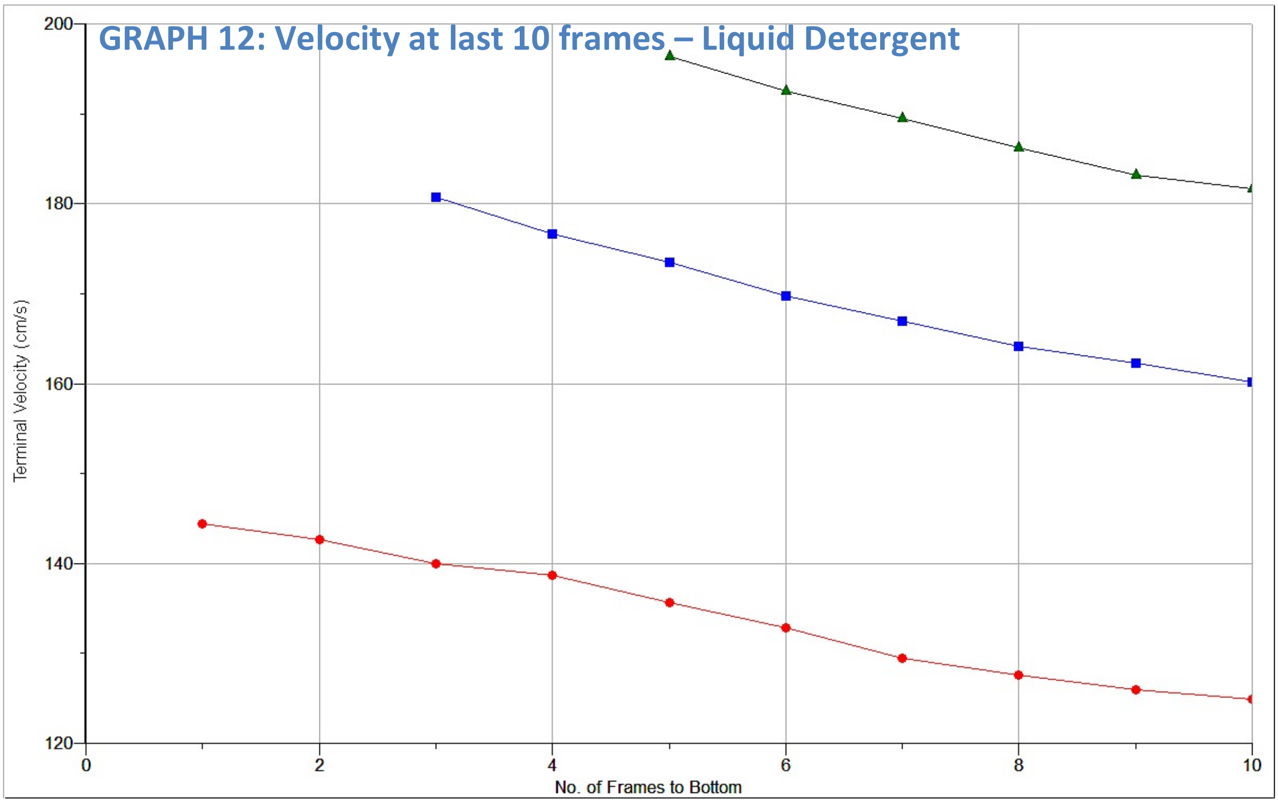
The most significant error is the camera frame rate. The camera frame rate used in this experiment was 25fps. This lead to blurring at high velocity, which increased uncertainty and error. Could be improved by higher frame rate camera.

Systematic Error and Random Error: Sphere not reaching terminal velocity and increased turbulent forces

At high temperatures, the sphere falls through the liquid at higher velocities. This leads to two systematic errors. The first is that the sphere does not have the chance to reach terminal velocity, as the experiment container is not long enough, hence does not give it enough time (See Graphs Below). A longer experiment container or a more viscous liquid would fix this, where velocities would be lower.

In addition to that, turbulent forces increase mainly due to increase in velocity (Equation for Reynolds Number).


GRAPH 11: Velocity at last 10 frames – Castor Oil
Data Set | 80/C
Data Set | 85/C
Data Set | 90/C
Data Set | 95/C
Terminal Velocity
200
150
100
0
2
4
6
8
10
No. of Frames to Bottom


GRAPH 12: Velocity at last 10 frames – Liquid Detergent
Terminal Velocity (cm/s)
200
180
160
140
120
0
2
4
6
8
10
No. of Frames to Bottom

The graphs above show that terminal velocity was not reached even until the last 10 frames (terminal velocity would be represented by a straight line, parallel to the x-axis).

Larger spheres would lead to great unreliability and uncertainty because they are heavier and hence would fall at greater velocity. This might lead to the sphere not reaching terminal velocity and significant blur in frames of photos. Additionally, turbulent forces affect larger spheres more that smaller spheres.

Random Error: Unconsidered Forces

All viscosities seem to be lower than literature values. This is probably due to other resistive forces not considered, for example *Newtonian Drag.* This drag is significant but otherwise always present for higher velocities, when laminar flow is not present.

Random Error: Parallax Error

Parallax error could have been committed when reading temperature readings. The parallax error caused by the camera (since whole cylinder had to fit in the camera frame) was considered (see camera calibration above). This estimation leads to uncertainty. It could have been avoided by either conducting more accurate processing, or by using a wide-lens camera.

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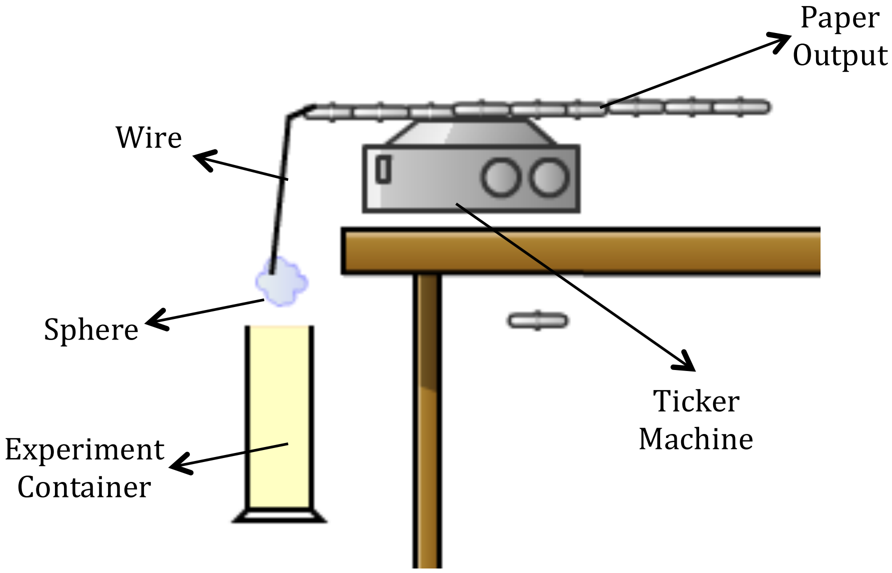
APPENDIX 1

Another, method that could be more accurate because it eliminated the use of a camera is by using Atwood's machine (ticker machine). In this process, the paper output of a ticker machine I attached to a sphere with a very thin wire, as in the diagram.



Since the ticker machine marks the paper in regular intervals, the distances between every next spot can be divided by the time interval in order to get the velocity (in the process above, this was calculated by taking the distance covered on camera divided by time interval between frames).

This method eliminates the use of camera; Hence any parallax error and other uncertainties caused due to blurring would be eliminated. Additionally, since the marks are made by a machine, at perfect intervals, error is eliminated (error of estimation and conversion from pixels to centimeters on camera).

However, some errors remain. Firstly, the sphere would still not reach terminal velocity at high temperatures ad the experiment container is the same length. This method lends itself to some other uncertainties however, which will have to be considered. First , the friction of movement of the paper in the ticker machine will have to be quantified, as it will result in an additional upward force, other than the force of buoyancy.

Additionally, the wire will add its own tension force and disturb the motion of fluid around the sphere. This could have unexpected effects leading to random error.

APPENDIX 2 – Program Code for Viscosity

```
import java.util.*;
public class viscositycoeff
{
    public static void main (String []args)
    {
        Scanner scan=new Scanner(System.in);
        System.out.println("Kindly enter all values in SI units");
        System.out.println("The value of gravity used is: 9.81 m.s^-2");
        System.out.println("Enter the number of liquids");
        int liquids=scan.nextInt();
        System.out.println("Enter the radius of sphere");
        double radius=scan.nextDouble();
        System.out.println("Enter the density of sphere");
        double density_sphere=scan.nextDouble();
        double t[]=new double[1000];
        double v[]=new double[1000];
        int arraycounter=0;
        for (int a=0; a<liquids; a++)
        {
            System.out.println("Liquid Number: "+(a+1));
            System.out.println("Enter the density of the liquid");
            double density_liquid=scan.nextDouble();
            double x=0;
            int counter=0;
            System.out.println("Enter the Temperature followed by the Terminal Velocity. Enter -1 when done.");
            while (x>=0)
            {
                counter=counter+1 ;
                System.out.println("Trail No.: " +counter);
                double temp=scan.nextDouble();
                double tv=scan.nextDouble();
                t[arraycounter]=temp;
                v[arraycounter]=tv;
                double visccoef=(2*(radius*radius)*9.81*(density_sphere-density_liquid))/(9*tv);
                if (tv>=temp)
                {
                    x=temp;
                }
                if (temp>=tv)
                {
                    x=tv;
                }
                arraycounter=arraycounter+1;
            }
            t[arraycounter]=-1;
            v[arraycounter]=-1;
            arraycounter=arraycounter+1;
        }
        int printcounter=0;
        for (int a=0; a<liquids; a++)
        {
            System.out.println("Liquid Number: "+(a+1));
            while (t[printcounter]!=-1)
            {
                System.out.println(+t[printcounter] + "  " +v[printcounter] );
                printcounter=printcounter+1;
            }
            printcounter=printcounter+1;
        }
    }
}
```

This is the entire code used for calculating viscosity of all the liquids. The program takes input once for radius and density of sphere. Hence, if radius/density changes, the program has to be run several times for each. For each liquid, the density is input. Then the user inputs all temperature and terminal velocity values for that liquid, repeated for each liquid. At the end, the temperature and viscosity values are output together.

APPENDIX 3 – Program Code for Reynolds's Number

This is the Java code with class "reynolds" which was used for automatic computation of the Reynolds's number.

The variable type used is "double", which is a double precision, 64 bit memory type, which can hold 754 floating points. Since only up to 10 decimal points is used in this experiment, there is no additional uncertainty from using this code.

This class asks the user for input of al necessary data and calculates/outputs the Reynolds's number promptly. The number of liquids and radius of sphere used are only asked once and assumed to be kept constant. For each liquid, the density then has to be input. Finally, the terminal velocity and viscosity for each point is input and the program outputs the Reynolds's number for that point. In order to use the program for multiple spheres, the code has to be run again and again.

```
import java.util.*;
public class reynolds
{
    public static void main (String []args)
    {
        Scanner scan=new Scanner(System.in);
        System.out.println("Kindly enter all values in SI units");
        System.out.println("Enter the number of liquids");
        int liquids=scan.nextInt();
        System.out.println("Enter the radius of sphere");
        double sphere=scan.nextDouble();
        for (int a=0; a<liquids; a++)
        {
            System.out.println("Liquid Number: "+(a+1));
            System.out.println("Enter the density of the liquid");
            double density=scan.nextDouble();
            double x=0;
            int counter=0;
            System.out.println("Enter the Terminal Velocity followed by the Viscosity Coeeficient. Enter -1 when done.");
            while (x>=0)
            {
                counter=counter+1 ;
                System.out.println("Trail No.: " +counter);
                double tv=scan.nextDouble();
                double vc=scan.nextDouble();
                double rey=(2*sphere*tv*density)/vc;
                if (tv>=vc)
                {
                    x=vc;
                }
                if (vc>=tv)
                {
                    x=tv;
                }
                System.out.println("Reynolds's Number: " +rey);
            }
        }
    }
}
```

APPENDIX 4

This is a screenshot of excel, where calculations of values and percentage uncertainties was carried out. This was used along with the program above (the program above was modified to output data onto excel), in order for this process to be efficient.

BALL				CASTOR OIL	957			
Radius	0.0026125	Temp	1/Temp	Temp	Vel	ln(vel)	Visc	ln(visc)
Density	6025	298	0.0033557	25	12.2	2.50143595	0.61808167	-0.4811347
Gravity	9.81	303	0.00330033	30	16.1	2.77881927	0.46836003	-0.758518
	1.48788E-05	308	0.00324675	35	20.3	3.01062089	0.37145795	-0.9903196
		313	0.00319489	40	22.8	3.12676054	0.33072791	-1.1064593
		318	0.00314465	45	26.3	3.26956894	0.28671469	-1.2492677
		323	0.00309598	50	30.2	3.40784192	0.24968862	-1.3875406
		328	0.00304878	55	37	3.61091791	0.2037999	-1.5906166
		333	0.003003	60	46.2	3.8329798	0.16321637	-1.8126785
		338	0.00295858	65	54.2	3.99268091	0.1391254	-1.9723796
		343	0.00291545	70	64.6	4.16821441	0.1167275	-2.1479131
		348	0.00287356	75	75.1	4.31882056	0.10040741	-2.2985193
		353	0.00283286	80	87.4	4.47049528	0.08627685	-2.450194
		358	0.0027933	85	117.9	4.76983681	0.06395756	-2.7495355
		363	0.00275482	90	130.1	4.86830339	0.05796	-2.8480021
		368	0.00271739	95	154.6	5.04084114	0.04877488	-3.0205399

		GLYCEROL	1262			
Temp	1/Temp	Temp	Vel	ln(vel)	Visc	ln(visc)
298	0.0033557	25	9.2	2.21920348	0.77030346	-0.2609707
303	0.00330033	30	13.8	2.62466859	0.51353564	-0.6664359
308	0.00324675	35	18.3	2.90690106	0.38725638	-0.9486683
313	0.00319489	40	25.1	3.22286785	0.2823423	-1.2646351
318	0.00314465	45	35.5	3.5695327	0.19962794	-1.6113
323	0.00309598	50	47.6	3.86283276	0.14888218	-1.9046
328	0.00304878	55	63.2	4.1463043	0.11213278	-2.1880716
333	0.003003	60	81.7	4.403054	0.08674164	-2.4448213
338	0.00295858	65	101.2	4.61709876	0.07002759	-2.658866
343	0.00291545	70	118	4.77068462	0.06005756	-2.8124519
348	0.00287356	75	134	4.8978398	0.05288651	-2.9396071
353	0.00283286	80	159.7	5.07329706	0.04437565	-3.1150643
358	0.0027933	85	178.2	5.18290652	0.03976875	-3.2246738
363	0.00275482	90	198.5	5.2907891	0.03570172	-3.3325564

		LIQ DET	1021			
Temp	1/Temp	Temp	Vel	ln(vel)	Visc	ln(visc)
298	0.0033557	25	15.4	2.73436751	0.4834657	-0.7267749
303	0.00330033	30	20.7	3.0301337	0.3596798	-1.0225411
308	0.00324675	35	27.7	3.32143241	0.26878599	-1.3138398
313	0.00319489	40	37.8	3.6323091	0.19696751	-1.6247165
318	0.00314465	45	47.3	3.8565103	0.15740744	-1.8489177
323	0.00309598	50	60	4.09434456	0.12408953	-2.086752
328	0.00304878	55	75.3	4.32148013	0.09887612	-2.3138875
333	0.003003	60	124.7	4.82591085	0.05970627	-2.8183182
338	0.00295858	65	160.1	5.07579862	0.04650451	-3.068206
343	0.00291545	70	180.1	5.19351225	0.04134021	-3.1859196

At first the usage of 5 spheres was planned at each temperature, to not only provide more conclusive proof of the investigation above, but also to investigate additionally the effect of size and dimensions of sphere on viscosity. This idea however, was dropped because the heavier spheres did not reach terminal velocity even past 40℃ and were inconsistent, in that had very high Reynolds numbers.

This is the screenshot of the 5 spheres that were planned and their dimensions.

Sphere #	± 0.001 g **Mass**		± 0.005 mm **Diameter**		**Radius**		$\left[\frac{4}{3}\pi r^3\right]$ **Volume**		$\left[\frac{Mass}{Volume}\right]$ **Density**	
	value	%	value	%	value	%	value	%	value	%
1	0.45	0.22222222	5.225	0.09569378	2.6125	0.09569378	74.6891482	0.2870813	6024.97164	0.50930356
2	0.72	0.13888889	5.55	0.09009009	2.775	0.09009009	89.5112396	0.2702703	8043.68259	0.40915916
3	1.06	0.09433962	6.85	0.072992701	3.425	0.0729927	168.29466	0.2189781	6298.47672	0.31331772
4	2.08	0.04807692	8.44	0.059241706	4.22	0.05924171	314.793649	0.1777251	6607.50306	0.22580204
5	3.06	0.03267974	9.57	0.052246604	4.785	0.0522466	458.917306	0.1567398	6667.86796	0.18941955